Understanding Buckling Behavior and Using FE in Design of Steel Bridges

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ABSTRACT: This paper describes how Finite Element (FE) analysis can be used to predict buckling modes, highlighting methods which are practical for day-to-day use. It explores some criterion which might be used to identify if such behavior, including global buckling modes, should be of concern to the designer, drawing on recommendations including those in the recently published NCHRP Report 725 and Eurocodes. Use of FE in the determination of member resistances is also explored.

INTRODUCTION

Buckling affects slender flexural and compressive members. It is caused by the amplification of initial imperfections and of deflections due to lateral loads and is characterized by sudden loss of stiffness with little or no warning, potentially leading to catastrophic failure.

Finite Element (FE) analyses can be used to analyze members or whole structures, considering non-standard details, support conditions and load arrangements as necessary*. FE buckling analysis options fall into two categories: Elastic (Eigenvalue) buckling analyses and nonlinear analyses.

Several uses for Eigenvalue buckling analyses will be explored in this paper:

1.1 Elastic critical buckling stresses can be used, together with codified buckling curves, in the determination of member resistances.

1.2 The global stability of a girder system can be assessed using an elastic critical buckling moment (M_{crG}).

1.3 A global amplification factor (AF_G) can be used to assess the susceptibility of a structure to second-order effects.

1.4 Likely buckling modes are identified and may be visualized, resulting in better understanding of structural behavior.

Furthermore, some instances for which nonlinear buckling analyses are required will be identified. In broad terms, such analyses may be understood to allow:

2.1 Detailed consideration of second-order effects.

2.2 Investigation into any unexpected behavior highlighted by an Eigenvalue buckling analysis.

2.3 Consideration of structures & details which are outside the scope of the code.

2.4 Validation of member resistances derived from code rules, particularly when considering an unusual detail or unfamiliar clause.

2.5 Identification of more economical solutions in certain cases.

In order to gain these benefits and have confidence in the results obtained using FE methods, engineers need to understand the analysis options available, in the context of other existing methods, and adopt suitable checking procedures, such as the calculation of bounding values. This paper is intended to assist by giving such context and indicating some possible checks.

BEAM AND SHELL ELEMENTS

The finite element types referred in this paper are restricted to 3D beam and 3D shell elements.

Where a structural member has cross-sectional

* Since the basic principle of subdividing a structure into simple elements can be applied to structures of all forms and complexity, there is no logical limit to the type of structures that can be analysed and FE provides the most versatile method of analysing structures available at present, and, for many structures, the only practical method. FE is widely accepted in the industry and endorsed in AASHTO [4] article 4.4. A familiarity with basic FE concepts is assumed.

dimensions that are small by comparison to its length, the member can typically be represented using beam elements. Beam elements are then described by giving a direction and cross-sectional properties (A, I_{yy}, I_{zz}, I_{yz}, J, A_{sy}, A_{sz}). 3D beam elements support axial force, shears, moments and torsion and generally have 6 degrees of freedom. Beams can be formulated either as thick (including shear deformations) or thin (excluding shear deformations). The beams referred to in this paper are thick linear order (BTS3), semiloof (BSL4) and semiloof cross-section a.k.a. "fiber" beam (BXL4), with reference to [1].

Where a structural member has a thickness which is small by comparison to its plan area, the member can typically be represented using shell elements. Shell elements are described in terms of a reference plane, typically (though not exclusively) at the mid-surface of the structural member, and a thickness (t). 3D shell elements support in-plane forces, out-of-plane shears, flexural moments and twisting moments. They generally have 6 degrees of freedom, enabling them to be connected to 3D beam elements. Like beam elements, shells can be formulated either as thick or thin. The shells referred to in this paper are thick quadratic order (QTS8) and semiloof (QSL8) [1].

The selection of elements and use of sufficient numbers of elements is important in obtaining appropriate results and will be discussed in the context of several simple examples.

CRITICAL ELASTIC BUCKLING

Central to most engineers' understanding of buckling is the "Euler" buckling load, after Leonhard Euler, who derived the well-known expression for the load at which an ideal pin-ended strut will first become unstable and buckle if slightly perturbed from its equilibrium position:

$$P_e = \frac{\pi^2 \mathrm{EI}}{L^2} \tag{1}$$

The formula is commonly adjusted to take account of conditions other than pinned ends. However, in principle the formula is the solution to the second-order differential equation governing the lateral deflection (v) of the column in the absence of transverse loading [2]:

$$\mathrm{EI}\frac{d^2\mathrm{v}}{dz^2} + P\mathrm{v} = 0 \tag{2}$$

Due to initial out-of-straightness, residual stresses and other effects such as load-following, nominal member resistance is generally less than an Euler-type solution would predict*. However, this "critical elastic buckling load" is used in the calculation of member resistances and amplification factors and is therefore required by engineers on a day-to-day basis. Crucially, if the same problem is formulated numerically, an Eigenvalue solution can be obtained which corresponds to the Euler solution. The "critical elastic buckling load" obtained by Euler or by numerical means (e.g. FE analysis) may be used interchangeably.

EXAMPLE EULER STRUT

An Eigenvalue extraction from an FE model is used to obtain the critical elastic buckling load for a concentrically loaded pin-ended strut. Using FE analysis for a pin-ended strut is, of course, unnecessary since the solution is trivial, however, such an example can be used to identify some modeling considerations which apply equally to more complex structures.

PROBLEM DETAILS. L=18'. H-section with D=B=8", flange & web thickness = 0.5". A_g=11.5in^2, I_{zz}=42.74in^4. Steel E=29000ksi.

By hand calculations, P_e=262.2kip.

Results from FE analyses are shown in Table 1.

Elements in height	Thick beam	Semiloof beam	Thick shell
4	290.8	262.1	260.8
8	268.3	262.2	259.9
16	263.1	262.2	259.9
32	261.8	262.2	259.9

Table 1. Critical elastic buckling load for example strut (kip)

Several things may be noted from this example:

BOUNDARY CONDITIONS. Naturally it is very important that supports represent the intended structure. However, a common modeling error leading to numerical difficulties in a problem of this sort is the omission of a torsional support, which is

* For slender plates in compression, member resistance may be greater than the elastic critical buckling stress because of post-buckling behaviour; see below and [3] Fig 4.6 (page 134)

required to prevent bodily rotation of the strut about its own longitudinal axis.

ELEMENT CHOICE AND REFINEMENT. By nature, all FE analyses are a numerical approximation of the real (or potentially real) structure. The results will only be accurate if the mesh is defined in such a way as to simulate the change in load effects across the structure effectively. Using a small number of elements may produce results which are significantly inaccurate. Moreover, a coarse mesh may produce results that are unconservative for design purposes. Here it is seen (Table 1) that the thick beam element results converge (with increased numbers of elements) from above, while the semiloof beam element results converge from below, reaching a satisfactory accuracy with far fewer elements. This illustrates the importance of shape functions. Engineers should familiarize themselves with the elements available in their specific FE software system and/ or experiment with different element types if possible.

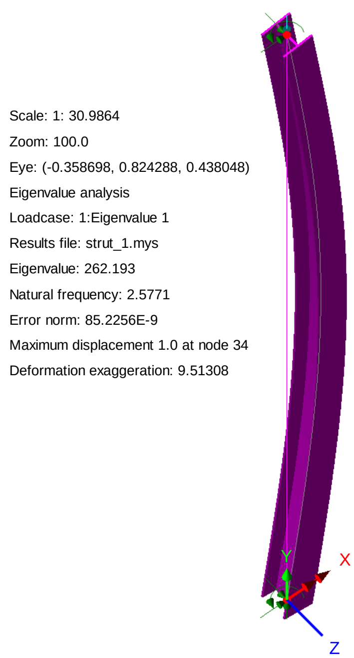

Figure 1(a). Critical elastic buckling mode for strut by FE method (beam model)

FLANGE LOCAL BUCKLING. Contrary to a beam element model, a shell element model allows plate local buckling modes to be identified and visualized in addition to the "global" buckling modes. In order to model the same (pin-ended) system using shells, a rigid link constraint equation is used at each end of the strut. In this particular case the lowest buckling mode is the expected half-wave and for a problem like this, beam elements are perfectly sufficient (Figure 1(a)), and computationally far more efficient. For other problems, shell elements may well be necessary to capture the full behavior of the structure adequately (Figure 1(b) and (c)).

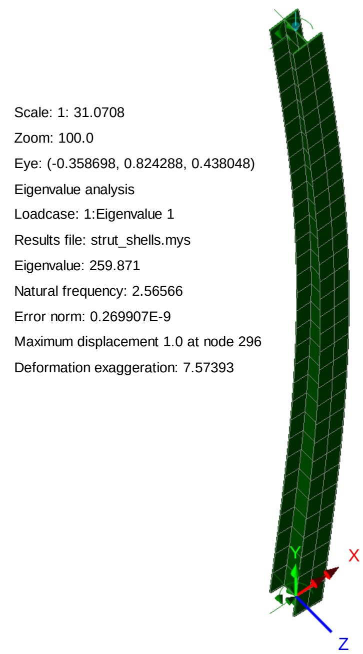

Figure 1(b). Critical elastic buckling mode for strut by FE method (shell model)

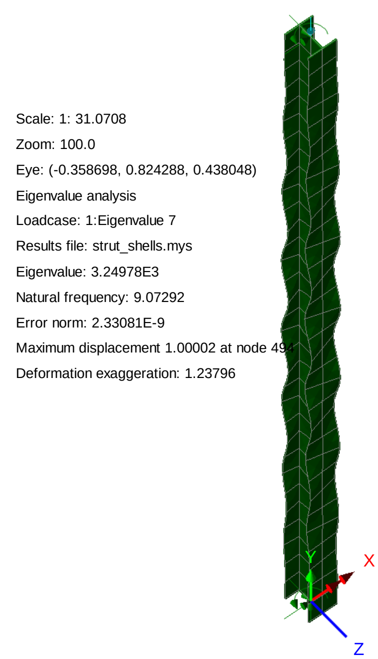

Figure 1(c). Critical elastic flange buckling mode (shell model)

COMPRESSIVE MEMBER RESISTANCE

In principle, compressive member resistance checks [4, 5] are based on identifying the slenderness (λ) of a member in terms of the critical elastic buckling load (P_e) and an equivalent nominal yield resistance, (P_o) *:

$$\lambda = \sqrt{\frac{P_o}{P_e}} \quad (3)$$

Design resistance is then derived from a "critical load curve" (see Figure 2 below), where the nominal compressive resistance (P_n) for a given value of λ† may be obtained from the corresponding y-axis value. Such critical load curves are empirically based, taking into account residual stresses and initial out-of-straightness [3] (pg 52ff).

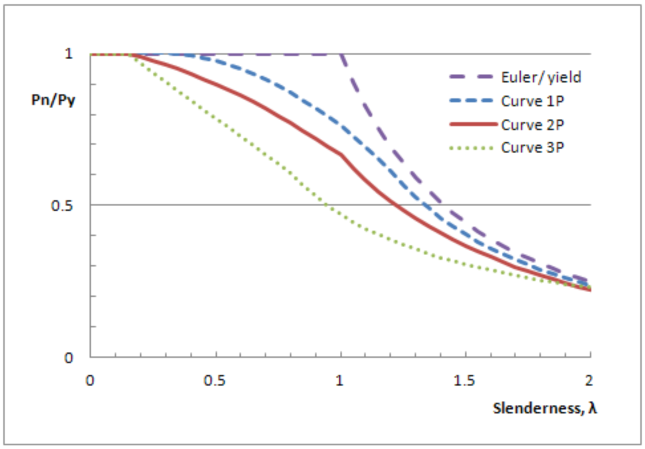

Figure 2. SSRC column strength curves, after Galambos [3]

AASHTO [4] equations 6.9.4.1.1-1 and 6.9.4.1.1-2 represent a curve that is essentially the same as the SSRC column strength curve 2P of [3], illustrated in Figure 2 above. In this sense, the applicability of the codified equations is limited to certain classes of column as identified by Bjorhovde [3] (pg 54), and an initial out-of-straightness of L/1500 is assumed – ref [3] (pg 59), AASHTO [4] C6.9.4.1.1. The Eurocode (EN1993-1-1 [5] clause 6.3.1.2) appears to be - in this area - more generalized, offering a range of curves represented by a reduction factor (χ), calculated from not only the slenderness (λ) but also an imperfection factor (α) appropriate to the type of section.

AASHTO gives several formulae for critical elastic buckling resistance (P_e) in section 6.9.4.1, which must be selected by virtue of the section type and potential buckling modes, and which are based on effective lengths (article 4.6.2.5), since the underlying critical load curve is for a pin-ended strut. In the Eurocode, such formulae are not given; it is anticipated that P_e can be determined by the engineer on the basis of formulae commonly available in textbooks - or by numerical methods.

The design resistances in both AASHTO and the Eurocode are thus based upon the critical elastic buckling load (P_e) which may be determined using an Eigenvalue extraction from a finite element (FE) analysis – as AASHTO [4] C6.9.4.1.1 describes it: "a refined buckling analysis".

However, moving beyond Eigenvalue analyses and codified approaches, a nonlinear analysis can provide an alternative means for assessing failure loads.

EXAMPLE FAILURE LOAD OF STRUT

A nonlinear analysis of the strut in the example above is carried out to compare with a member resistance derived from AASHTO. Nonlinear FE analyses can incorporate geometric nonlinearity (large displacements), material nonlinearity (in this case, yielding) and boundary nonlinearity (such as lift-off). Any nonlinear buckling analysis will be based on geometric nonlinearity but may also require other nonlinearities to be incorporated in order to capture the true behavior of the structure.

MEMBER RESISTANCE CALCULATION. As previously noted, P_e=262.2kip. Being made up of nonslender plates according to AASHTO article 6.9.4.2.1, $P_o=F_yA_g$=50*11.5=575kip.

Thus from article 6.9.4.1.1, P_e/P_o=0.456 and P_n=230kip.

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* P_o incorporates a reduction factor (Q in AASHTO [4] article 6.9.4.2) to take account of plate buckling. In EN1993-1-1 [5] clause 6.3.1.1(3) the same phenomenon is handled by the use of a reduced effective area (A_{eff}).

† For a strut without local buckling, it may be seen that this definition of slenderness can be reduced to a material constant × effective length / radius of gyration, more familiar to many engineers.

$$\lambda = \sqrt{\frac{P_o}{P_e}} = \sqrt{\frac{F_yA}{\frac{\pi^2 \mathrm{EI}}{(kL)^2}}} = \frac{1}{\pi}\sqrt{\frac{F_y}{E}}.\frac{kL}{r}$$

INITIAL IMPERFECTIONS. The strut as modeled is perfectly straight, symmetrical and concentrically loaded. In order for any geometrically nonlinear analysis to indicate buckling, an initial imperfection must be incorporated. As noted above, the AASHTO column design curves assume an imperfection of L/1500 but also incorporate an allowance for residual stress. AASHTO does not appear to give further guidance on imperfections for use in second-order analyses. Since other sources suggest larger initial imperfections*, the sensitivity of the analysis to the magnitude of initial imperfections may be of interest.

GEOMETRICALLY NONLINEAR ANALYSIS. Based on the prior Eigenvalue analyses, 4 semiloof beam elements are used. These elements support geometrically nonlinear analysis with the Total Lagrangian formulation. A geometrically nonlinear analysis with an initial imperfection of L/1500 is carried out; the initial shape is created by inheriting the deformed shape of the lowest buckling mode illustrated in Figure 1(a) above, factored to 216/1500=0.144". The buckled shape and load/ displacement curve are shown in Figure 3. The stresses illustrated in Figure 3(a) are appropriate to the change in slope of the graph (P=237kip) and it is noted that stresses exceed 50ksi. Furthermore the load/ displacement curve indicates continued strength in the strut, up to 261kips (albeit with uncontrolled displacement). This preliminary analysis indicates that material nonlinearity plays a part in the failure of the strut and must be incorporated.

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* EN1993-1-1 [5] suggests using the shape of the elastic critical buckling mode as an imperfection when second-order analysis is used (see clause 5.3.4) with the amplitude based on the section in question (see Table 6.2 and Table 5.1 in conjunction). Broadly speaking, the imperfections are of order span/150, or span/300 for heavy bridge sections if LTB is concerned. These values - significantly greater than expected fabrication tolerances - incorporate an allowance in lieu of residual stresses.

BS5400-3:2000 [11] clause 9.12.1 recommends that initial imperfections for use in nonlinear buckling analyses should be 1.5 times the relevant tolerances given in BS5400-6:1999 Table 8. This leads, broadly speaking, to imperfections of order 1.5*L/1000 or 5mm (whichever is greater) for a bridge girder.

AISC 303-10 [12] gives fabrication tolerances (section 6.4) which (for spans >30ft) come down to L/1000 or 1/8", whichever is greater, and erection tolerances which, are broadly of the order L/500. Using the British 1.5 factor, initial imperfections of the order L/300 seem reasonable, in agreement with the Eurocode.

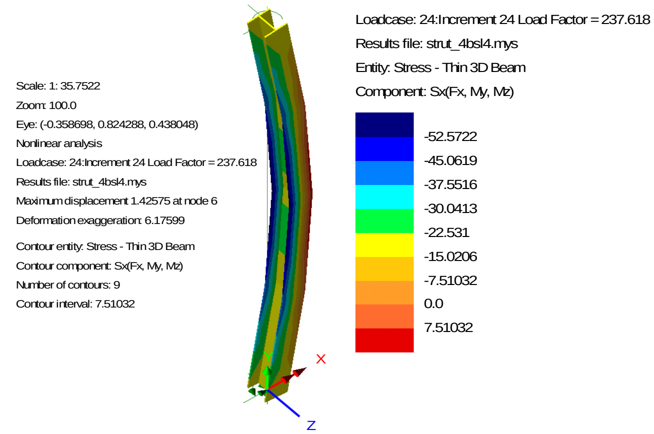

Figure 3(a). Buckling of strut in preliminary GNL analysis

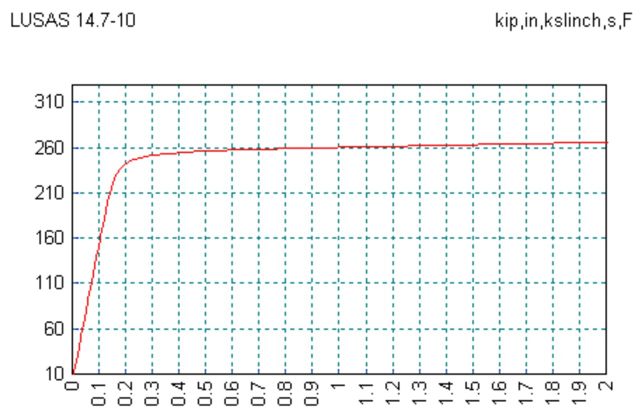

Figure 3(b). Load/ displacement of strut in preliminary GNL analysis

MATERIAL NONLINEARITY. In order to facilitate gradual through-section yielding, the semiloof beams are switched to semiloof cross-section beams. Yield is set at 50ski and tensile strength taken as 65ksi, with a 21% elongation, in a stress potential (Von Mises) material model. The load/ displacement curve is shown in Figure 4.

The effect of the magnitude of initial imperfections was investigated by re-running the full nonlinear beam model with a range of imperfections as shown in Table 2 below.

The maximum load associated with an initial imperfection of L/1500, P_n=232kip, agrees closely with the AASHTO member resistance calculated.

By way of validation, a shell model using semiloof shell elements, the same material properties and an initial imperfection of L/1500 was analyzed, giving a maximum load of P_n=237kip.

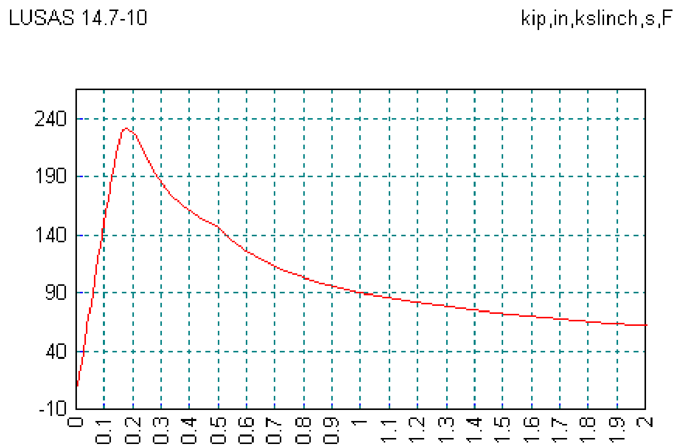

Figure 4. Load/ displacement of strut in full NL analysis

Magnitude of initial imperfection	Maximum load, P_n (kips)
L/1500	232
L/1000	223
L/500	202
L/300	184

Table 2. Effect of initial imperfection on Maximum load in strut

LONG PLATES IN COMPRESSION

The equivalent nominal yield resistance of a compression member incorporates a reduction to take account of plate buckling (AASHTO [4] article 6.9.4.1). Local flange buckling (AASHTO [4] article 6.10.8.2.2) also makes consideration of such effects. In principle the approach is similar to that described for columns above, in this case relying on a slenderness (λ) defined in terms of the critical elastic buckling stress (F_{cr}) and yield stress (F_y):

$$\lambda = \sqrt{\frac{F_y}{F_{cr}}} \tag{4}$$

Design resistance may again be derived from a critical load curve, similar to Figure 2 – for example, the buckling curves of [3] (pg 134) *. The reduction factors of AASHTO [4] article 6.9.4.2.2 reflect such a form.

The elastic critical buckling stress of a plate is therefore of importance to design engineers, and the general expression given in AASHTO [4] C6.9.4.2.1 and [3] (pg 126) is:

$$F_{cr} = k\frac{\pi^2 \mathrm{E}}{12(1-\nu^2)(b/t)^2} \tag{5}$$

Here, the buckling coefficient (k) is a function of loading and support conditions. Values for k may be obtained from texts for a limited number of conditions – ref [3] (Fig 4.2 pg 127), for example, where both long edges are simply supported, k=4.0.

As an alternative, F_{cr} may be determined using an Eigenvalue extraction from an FE analysis. Any such value of F_{cr} may be of direct use, else values of k may be back-calculated if required†.

EXAMPLE ELASTIC BUCKLING OF A LONG PLATE

An Eigenvalue extraction from an FE model is used to obtain the critical elastic buckling load for a long simply supported plate. As for the strut example, using FE analysis is unnecessary in this case, however, the example serves as a benchmark to help build confidence when more complex structures are considered.

PROBLEM DETAILS. L=120”, b=12”, t=0.5”. Steel E=29000ksi, v=0.3. The plate simply supported and end-loaded.

By hand calculations, using k=4.0, F_{cr}=182ksi

Results from FE analyses are shown in Table 3.

Element size	Semiloof shell (QSL8)
3”	171.8
2”	177.6
1”	181.0

Table 3. Critical elastic buckling stress for example plate (ksi)

It is noted that the semiloof shell converges to the expected result with increased mesh refinement,

* Unlike columns, the buckling resistance of slender plates may be greater than the elastic critical buckling stress because of post-buckling behavior. This is because plates have a significant post-buckling reserve of strength (dependent upon boundary conditions), whereas columns are essentially in a state of neutral equilibrium after elastic buckling

† It should be noted that the “classical” buckling coefficient k in AASHTO [4] C6.9.4.2.1 and “tabulated” buckling coefficient k in AASHTO eqn 6.9.4.2.1-1 are not the same, and are related as follows: $k_{tabulated} = \pi\sqrt{\frac{k_{classical}}{12(1-\nu^2)}}$

from the "safe side".

FLEXURAL MEMBERS

In flexural members, there are several buckling modes which are significant. Flange local buckling (FLB) is covered explicitly in AASHTO [4] article 6.10.8.2.2. Lateral torsional buckling (LTB) is considered in article 6.10.8.2.3, and takes the form of checks on sections between bracing locations, which are taken as points of fixity, with the unbraced length (L_b) being critical to the design. Web local buckling is not covered explicitly in section 6.10.8, being effectively precluded by the web proportion limits in article 6.10.2.1 – this presents difficulties when rating existing members that do not meet the requirements of that article.

As illustrated in Figure 5 (also described well in AASHTO [4] Fig C6.10.8.2.1-1 and [3] Fig 5.1, page 193), design resistances based on FLB and LTB are implicitly based upon buckling curves not dissimilar to those described earlier in this paper.

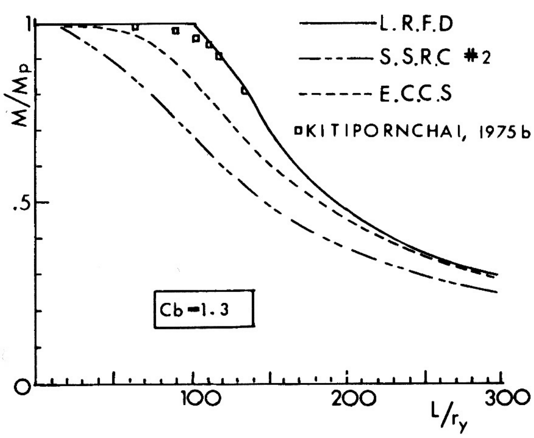

Figure 5. Flexural buckling curve from [3]*

In principle, slenderness is again defined in terms of elastic critical stress, although, working on an assumption of effective bracing, it can be expressed simply using the ratio L_b/r_y. Beyond a limiting unbraced length ($L_b>L_r$), the member resistance is assumed to be given by the elastic critical stress - as given in AASHTO [4] eqn 6.10.8.2.3-8:

$$F_{cr} = \frac{C_b R_b \pi^2 \mathrm{E}}{(L_b/r_1)^2} \qquad (6)$$

This is noted (C6.10.8.2.3) to be an accurate to conservative simplification, based on the theoretical solution for a doubly-symmetric I-section for the case of constant moment (C_b = 1.0) assuming the St Venant torsional constant (J) to be zero, and an alternative expression is offered in eqn A6.6.3-8 which may be applied in certain circumstances.

In the check on member resistances in AASHTO [4] (article 6.10.8.1) a flange lateral bending stress is required. By reference to article 6.10.1.6, the flange lateral bending stress for larger unbraced lengths is given by:

$$f_l = \left(\frac{0.85}{1-\frac{f_{bu}}{F_{cr}}}\right) f_{l1} \geq f_{l1} \qquad (7)$$

Once again the elastic critical buckling stress (F_{cr}) is required for the calculation. In connection with this, 3D FE analysis is, in broad terms, needed if flange lateral bending stresses and cross-frame forces are to be determined with good accuracy - refer to NCHRP [6] Report 725 Appendix B Table 3.1.

A wholly different approach is taken in the Eurocode (EN1993-1-1 [5] clause 6.3.2.2) for flexural members – an approach consistent with that for compressive members – defining slenderness as:

$$\lambda_{LT} = \sqrt{\frac{M_y}{M_{cr}}} \qquad (8)$$

With this in hand, Eurocode design curves are implemented by way of a reduction factor (χ_{LT}), calculated from λ_{LT} but also an imperfection factor (α_{LT}) appropriate to the type of section.

There seems to be no reason why F_{cr} or M_{cr} may not be determined using an Eigenvalue extraction from an FE analysis, as for compressive members. This would be beneficial since several assumptions implicit to the AASHTO formulae might be eliminated, such as those associated with web proportion limits. Furthermore, the engineer may gain a better understanding of structural behavior by virtue of being able to visualize the buckling modes rather than "blindly" calculating resistances using formulae. Such an approach is implicit to the Eurocode.

GLOBAL BUCKLING BEHAVIORS

AASHTO LTB checks currently consider only the member resistance of individual girders controlled by the distance (L_b) between brace points. In certain situations – particularly during construction of composite bridge decks while loads are carried by girders prior to the hardening of the concrete deck –

* Fig 5.13, pg 213. Reproduced by kind permission of the author.

the braced girder system may be susceptible to a lower global buckling mode over the span length (L_s).

Yura et al [7] proposed a simplified expression giving a reasonable estimate of the global buckling capacity to help engineers determine if global buckling is a concern:

$$M_{crG} = C_b \frac{\pi^2 sE}{L_s^2} \sqrt{I_{ye} I_x} \tag{9}$$

As for the AASHTO LTB formulae (6 above), expression (9) is based on the theoretical solution for a doubly-symmetric I-section and a moment gradient modifier (C_b) is required. Yura et al [7] offer adjustments for singly symmetric sections and for consideration of up to 4 girders but the approach necessarily has limitations such as the individual girders being assumed to be prismatic and all girders being assumed to have the same cross-section. NCHRP Report 725 [6] indicates that the engineer may need to exercise judgment in applying the equation.

As for other elastic critical buckling problems, M_{crG} may be alternatively determined using an Eigenvalue extraction from an FE analysis.

However determined, M_{crG} would be an unconservatively high assessment of the strength of the girder system, since it is based on the Euler assumptions of no imperfections etc. In particular, with any out-of-straightness (intended or otherwise), pre-buckling lateral deformations have a tendency to be self-reinforcing, described by NCHRP Report 725 [6] as "second-order lateral-torsional amplification of the displacements and stresses". The NCHRP report therefore recommends that the linear responses from a first-order analysis be multiplied by an amplification factor (10 below) – thus alerting the engineer to the presence of any significant global second-order effects:

$$AF_G = \frac{1}{1-\frac{M_{maxG}}{M_{crG}}} \tag{10}$$

NCHRP Report 725 [6] suggests that where $AF_G<1.1$ the influence of second-order effects may be neglected*. Where $AF_G>1.25$ during deck placement, NCHRP Report 725 [6] recommends use of a second-order (geometric nonlinear) 3D FE analysis.

* An identical approach and identical threshold is adopted in the Eurocodes [EN1994-1-1 [8] clause 5.2.1(3)] although it is expressed as a requirement for $\alpha_{cr} = M_{crG}/M_{maxG} > 10$.

EXAMPLE OF GLOBAL BUCKLING

PROBLEM DETAILS. The twin girder system of Widianto & Yura [9] is used, as illustrated in Figure 6 below. The twin girders span 170′ simply supported. Web panels are assumed to be 17′ in length (10 panels per girder) and bracing is provided at the supported ends and at alternate bays (34′ spacing). It is noted in Yura et al [7] that the behaviour of the system for three, four or five cross frames was almost identical.

In order to facilitate comparison with the results of Widianto & Yura [9], the girder system is subjected to four-point loading in order to generate a constant moment over the larger part of the girder, although it is noted that this loading regime is not representative of likely moment gradients during construction.

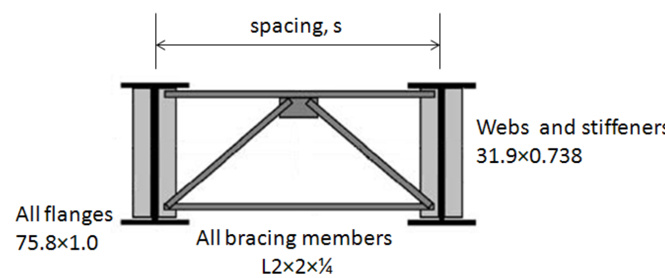

Figure 6. Example twin girder system

ELEMENT SELECTION. The girders are represented using semiloof shell elements for the web and semiloof beam elements for the top and bottom flanges, and web stiffeners. This is efficient and effective for modelling global and LTB modes, however may not be sufficient to capture local plate buckling in the flanges – where this is required, shell elements can be used throughout. The bracing is represented using semiloof beam elements.

RESULTS. Results are summarized in Table 4 below and illustrated in Figure 7.

Table 4 shows good agreement between FE results and hand calculation although some deviation is anticipated since the bracing details used by Widianto & Yura [9] are not available and the method of loading may be slightly different.

For this example, which uses a pair of identical prismatic, doubly symmetric girders with no skew and no plan curvature, the FE and simple rules provide comparable results, as should be expected. An FE solution allows the consideration of non-identical, non-prismatic, non-symmetric girders, as well as the consideration of skew and/ or plan curvature.

In any case, the Amplification Factors in Table 5, being >1.25, suggest a need to further investigate second-order effects through a geometric nonlinear 3D FE analysis.

Analysis type	**Girder spacing (in)**		
	80	**109**	**150**
From Eqn (9)	20.70	28.22	38.83
LUSAS, 50″ elements	20.13	26.78	35.60
LUSAS, 25″ elements	20.21	26.91	35.77

Table 4. Stress in extreme fibers* at global buckling of twin girder system, σ_{crG}

	Girder spacing (in)		
	80	**109**	**150**
AF_G from eqn (10)	1.59	1.38	1.26

Table 5. Amplification Factor based on global buckling of twin girder system, AF_G

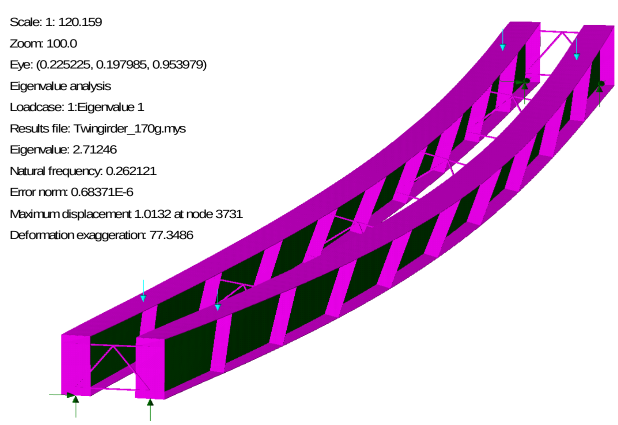

Figure 7. Global buckling mode from twin girder system

Interestingly, incorporating a modest skew of 20° reduced the amplification factor somewhat (that is, lower second-order effects were indicated). This is thought to be because of opposing torques at the supports. However, since lift-off could occur, a full nonlinear analysis should be used to properly consider stability issues.

Incorporating a modest plan curvature of radius 1200′ led to buckling in the bracing members at a lower load than buckling of the girder system, underlining the importance of these as structural members.

CONCLUDING REMARKS

The elements used in FE buckling analyses might comprise shells, beams or a mixture, on the understanding that plate local buckling effects cannot be identified using beam elements. The choice of elements should consider the shape function and mesh refinement should be checked.

Elastic critical buckling loads may be obtained from Eigenvalue buckling analyses. The elastic critical forces, moments or stresses may be used together with codified buckling curves, in the determination of member resistances. Importantly, buckling modes are identified and may be visualized, potentially resulting in better understanding of structural behavior than when calculations proceed "blind".

Second-order (nonlinear) analysis may be used to assess member resistances, and this may be appropriate when unexpected behavior has been highlighted by a prior Eigenvalue buckling analysis, when the structure or details are outside the scope of the code, or for validation when unusual detail or unfamiliar clauses are approached.

Initial imperfections need to be included in any nonlinear buckling analysis. Eigenvalue buckling mode shapes typically provide a suitable imperfect shape. The magnitude of the imperfection assumed has a significant effect on member resistances derived from nonlinear analysis - this underlines the limitation of codified buckling rules to members fabricated and erected to modern tolerances, and the possible need for nonlinear analysis to be used for members not meeting such standards.

It is perhaps not widely appreciated that the critical load curves which form the basis of member resistance calculations in design codes, such as those developed by Bjorhovde & Tall [10] were

* The extreme stress is calculated as $\sigma_{cr} = {}^{\alpha_{cr} M}/_{Z}$ where M is the applied moment (20,400kip.in), Z the section modulus (105,251/38.638=2724in^3) and α_{cr} the Eigenvalue buckling factor from the FE analysis.

themselves based on computer analyses - validated against physical tests - rather than a statistical analysis of data purely from physical tests. Thus, codified member resistance checks largely offer the practicing engineer access to the results of numerical analyses undertaken by researchers - perhaps 40 years ago - by way of slenderness and other parameters which may be calculated by hand. Ironically, as compared to their 1970's research counterpart, the practicing engineer of 2013 typically has a computer of considerably greater power, loaded with software of considerably greater sophistication at their disposal. With this in mind it is perhaps not unreasonable to suggest that engineers could consider using numerical methods which take into account initial imperfections, load-following and (potentially) yielding – nonlinear analyses – to assess member resistances.

Elastic critical buckling loads can also be used to investigate the global stability of a girder system and its susceptibility to second-order effects. While simple rules exist for girder systems meeting particular criteria, an FE Eigenvalue buckling analysis allows the consideration of non-identical, non-prismatic, non-symmetric girders, and those with skew and/ or plan curvature.

Second-order (nonlinear) analysis is recommended for structures with a large amplification factor (AF_G) by NCHRP Report 725 [6], AASHTO [4] clause 4.5.3.2, EN1994-1-1 [8] clause 5.2.1(3) etc, or for which lift-off may occur. Such analyses can be readily undertaken, based upon the same analytical models constructed for Eigenvalue buckling analyses.

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