

CUSTOMER SUPPORT NOTE

Viscous Transmitting and Bicanic Boundary Conditions

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This support note is issued as a guideline only.



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1. Introduction

This support note considers the application of boundary conditions for a transient dynamic soil structure interaction analysis using joint elements, and an alternative method of applying the input motion relative to the incoming seismic wave that negates the need for deconvolution [B1].

The approaches described are the application of Viscous Transmitting Boundaries using joint elements and Bicanic Boundary Conditions (BBC) applied at the base of the model.

Previous experience of using joint elements is assumed, however there are many pages in the User Area of the LUSAS website that provide further guidance

<http://www.lusas.com/protected/index.html>. See also the **Modeller Reference Manual>Chapter 5 : Model Attributes>Meshing>Joint/Interface Element Mesh** [L1], which describes how to assign the mesh and gives a case study example.

2. Description

For a transient dynamic soil-structure interaction analysis, let us first consider the effect of the following simplistic approach to the application of boundary conditions in a model which merely represents a small region of a freely extending mass of soil and rock as shown in Figure 1.

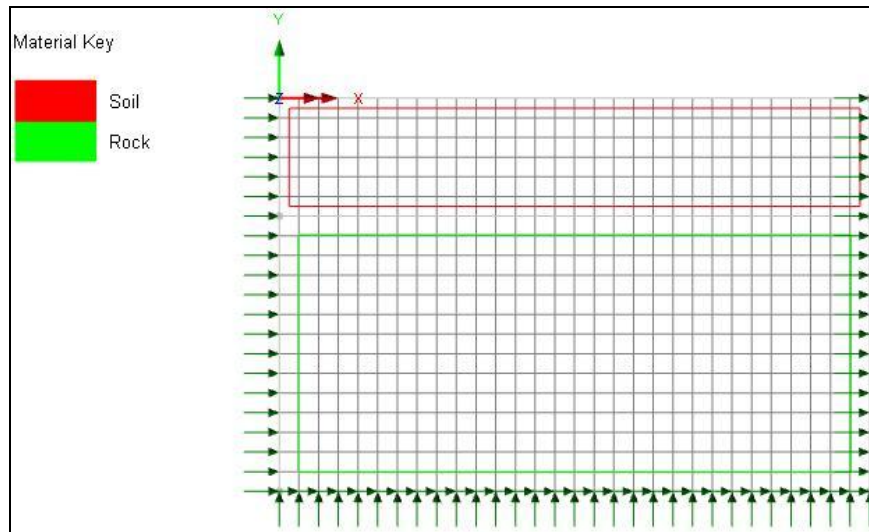


Figure 1 A Simplistic Approach to the Application of Boundary Conditions

The vertical artificial boundaries are modelled simply here with translation fixed in the X direction. In a transient dynamic analysis, such as a seismic analysis for example, incident waves at these boundaries are incorrectly reflected. Such boundaries are described as being Non-Transmitting Boundaries.

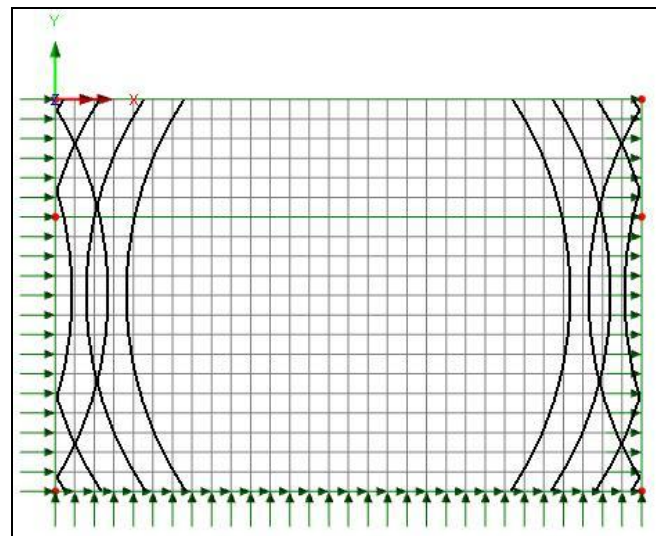


Figure 2 Incident Waves Reflected at Artificially Fixed Translation, Non-Transmitting Boundaries

The boundary conditions at the base of the structure are fixed in X and Y translation. These will also reflect incident waves and the depth to which the rock should be modelled truncated and supported needs to be decided. In a seismic analysis this fixed base is required if a "Support motion" approach is to be adopted whereby the excitation of an earthquake is applied using prescribed acceleration/velocity/displacement to this fixed base.

3. Viscous Transmitting Boundaries

A Viscous Transmitting Boundary is an artificial boundary which allows the modelling of a finite region of essentially unbounded soil, but where incident waves are transmitted rather than reflected.

The simple and original method of achieving this uses an equivalent dashpot. The characteristics of a dashpot can be considered analogous to the stress induced by propagation of elastic body waves.

The **shear stress** induced by an elastic shear wave (S-wave) is given by:

$$\tau = v \cdot \rho \cdot V_s \quad (3.1)$$

Where: v = particle velocity

V_s = Shear Wave Speed = $\sqrt{(G / \rho)}$

ρ = Density

G = Shear Modulus $E/(2 \cdot (1 + \mu))$

μ = Poisson's Ratio

The product ($\rho \cdot V_s$) is termed the "impedance" of the medium, Z_s .

The **normal stress** induced by an elastic pressure wave (P-Wave) is given by:

$$\sigma_p = v \cdot \rho \cdot V_p \quad (3.2)$$

Where: V_p = Pressure Wave Speed = $\sqrt{(M / \rho)}$

M = P-Wave Modulus = $E(1 - \mu)/((1 + \mu)(1 - 2\mu))$

A single joint element is used to achieve the required dashpot arrangement as shown in Figure 3. This is meshed between each node on the mesh boundary and a fixed node.

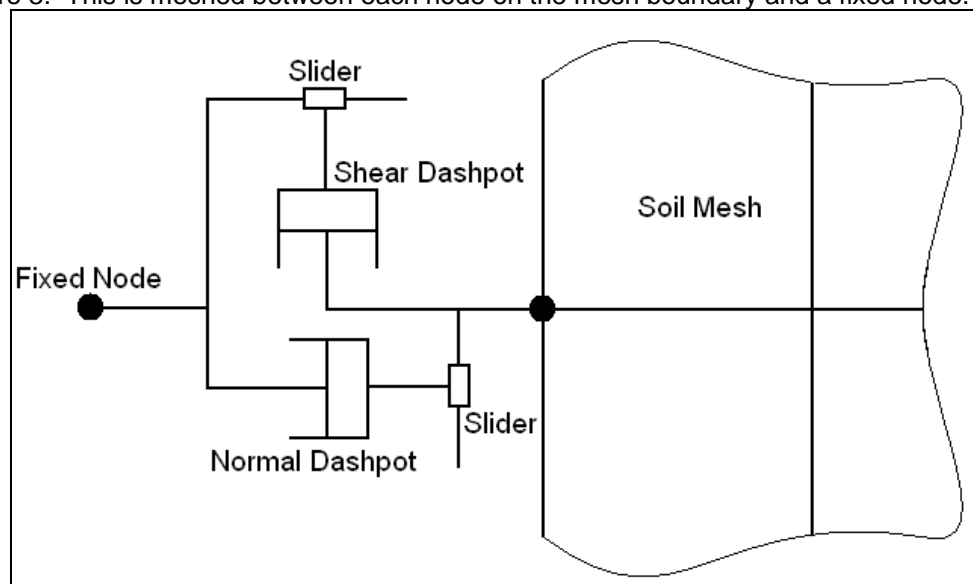


Figure 3 Viscous Transmitting Boundary

For a 2D Plane Strain model, the joint mesh is assigned between a pair of lines in LUSAS Modeller with a Local Coordinate system used to orientate the joint element axes. One line is assigned fixed supports and the other line is the boundary line on the surface that is assigned the soil mesh. This is illustrated in Figure 4 below, where the two lines are shown separated a distance apart whilst the mesh, material and supports are assigned to make the modelling clear. The lines can later be made unmergable and coincident once the pre-processing of the model is complete.

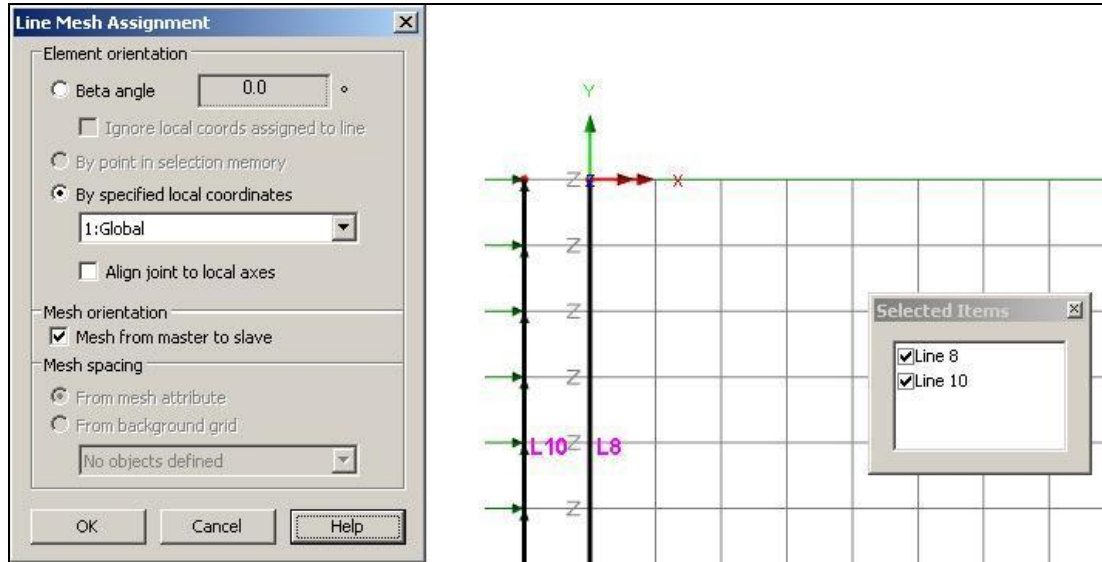


Figure 4 Assigning the Joint Mesh

The joint material used to define the normal and the shear dashpots is the “**General Properties**” joint material. The damping coefficient is determined from the Rayleigh Damping relation as follows:

$$C = \alpha.M + \beta.K \quad (3.3)$$

The mass M is set to 0 and the stiffness K is set to 1, thus the stiffness coefficient $\beta = C$. Rayleigh Damping defines damping as a proportion of mass and stiffness of the structure. This means that with the mass damping set to zero, the damping matrix will provide purely a 'relative damping' response [L2].

By comparison of equations 3.1 and 3.2 with equation 3.3 the damping coefficient C is taken as being equal to the impedance. Hence:

$$\text{In the normal direction to the boundary:} \quad \beta = C = Z_p = \rho.V_p \quad (3.4)$$

$$\text{In the shear direction to the boundary:} \quad \beta = C = Z_s = \rho.V_s \quad (3.5)$$

When specifying the damping for a joint either the Stiffness Rayleigh damping constant β can be entered or the viscous damping coefficient c .

If the input for the joint is to be entered in terms of Rayleigh Damping Parameters for a joint assigned to lines, then the joint stiffness value entered into the joint material in Modeller will need to be one multiplied by the thickness over which the joint acts. The entry in Modeller will be taken as stiffness per unit length for assignment to a line and will be tabulated in the

Datafile with suitable values per joint element. The Rayleigh damping constant inputs themselves are not affected by whether the joint is assigned to points or lines and will be tabulated in the Datafile by Modeller with the same value as entered in Modeller. The input for the Stiffness Rayleigh damping constant β is shown in Figure 5.

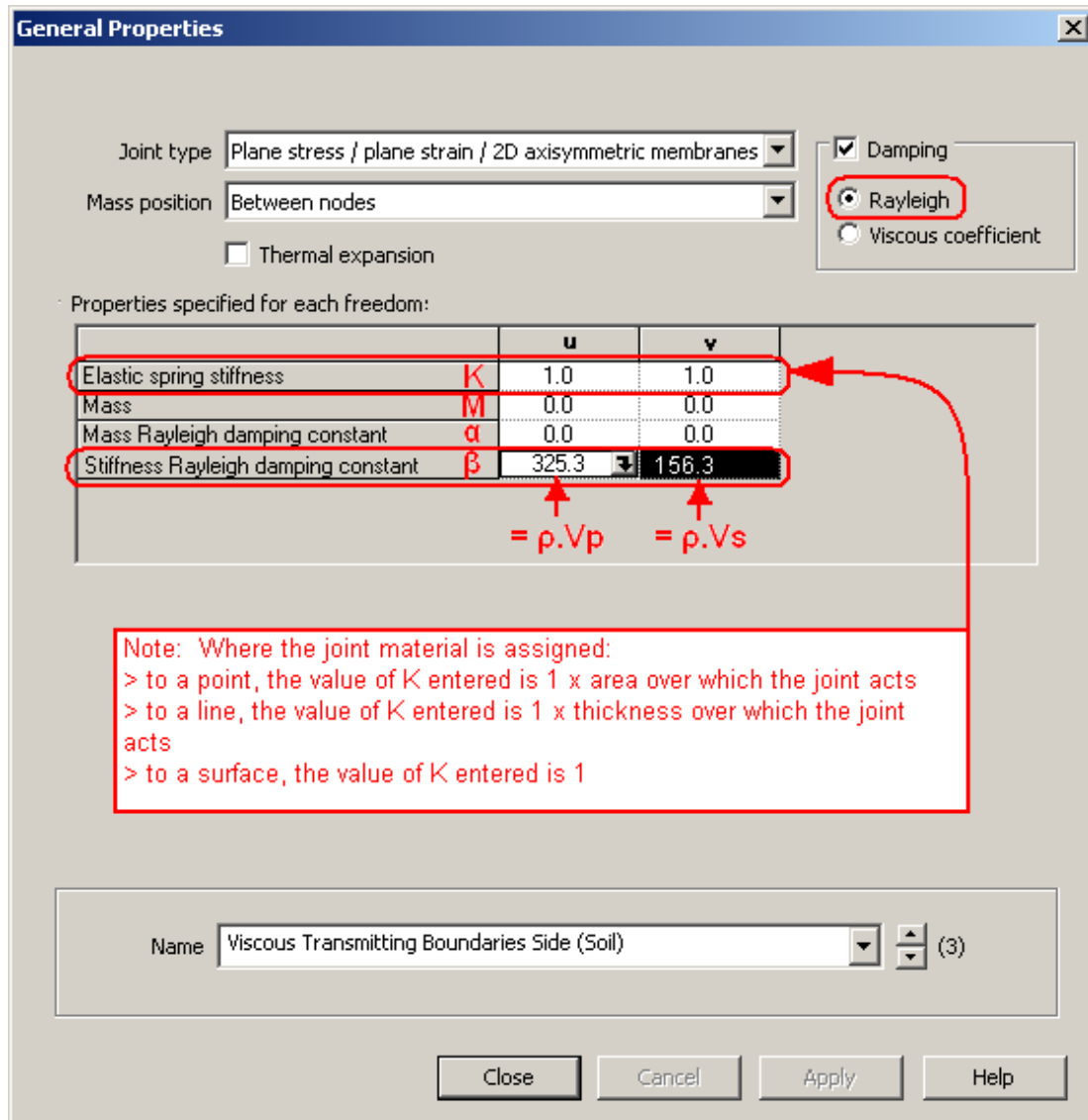


Figure 5 Joint Material - Entering Rayleigh Damping Parameters

For viscous damping (logarithmic decrement) the equation relating the damping force F_d to the velocity v is given as follows:

$$F_d = -c.v \quad \text{Eq. 3.6}$$

Where: c is the viscous damping coefficient ($C = c$ per unit area).

For a 2D plain strain model the thickness is unity, and for a joint assigned to lines the inputs are per unit length, so conveniently, the viscosity coefficient, c entered is effectively per unit area and so takes the same value as the stiffness coefficient β for this case. If defining the

viscosity coefficient directly, then the advantage is that the stiffness of the joint can be set to zero ($K=0$). Whilst a joint stiffness of 1 will often be negligible anyway, its significance will be unit dependent. The input for the viscosity coefficient is shown in Figure 6.

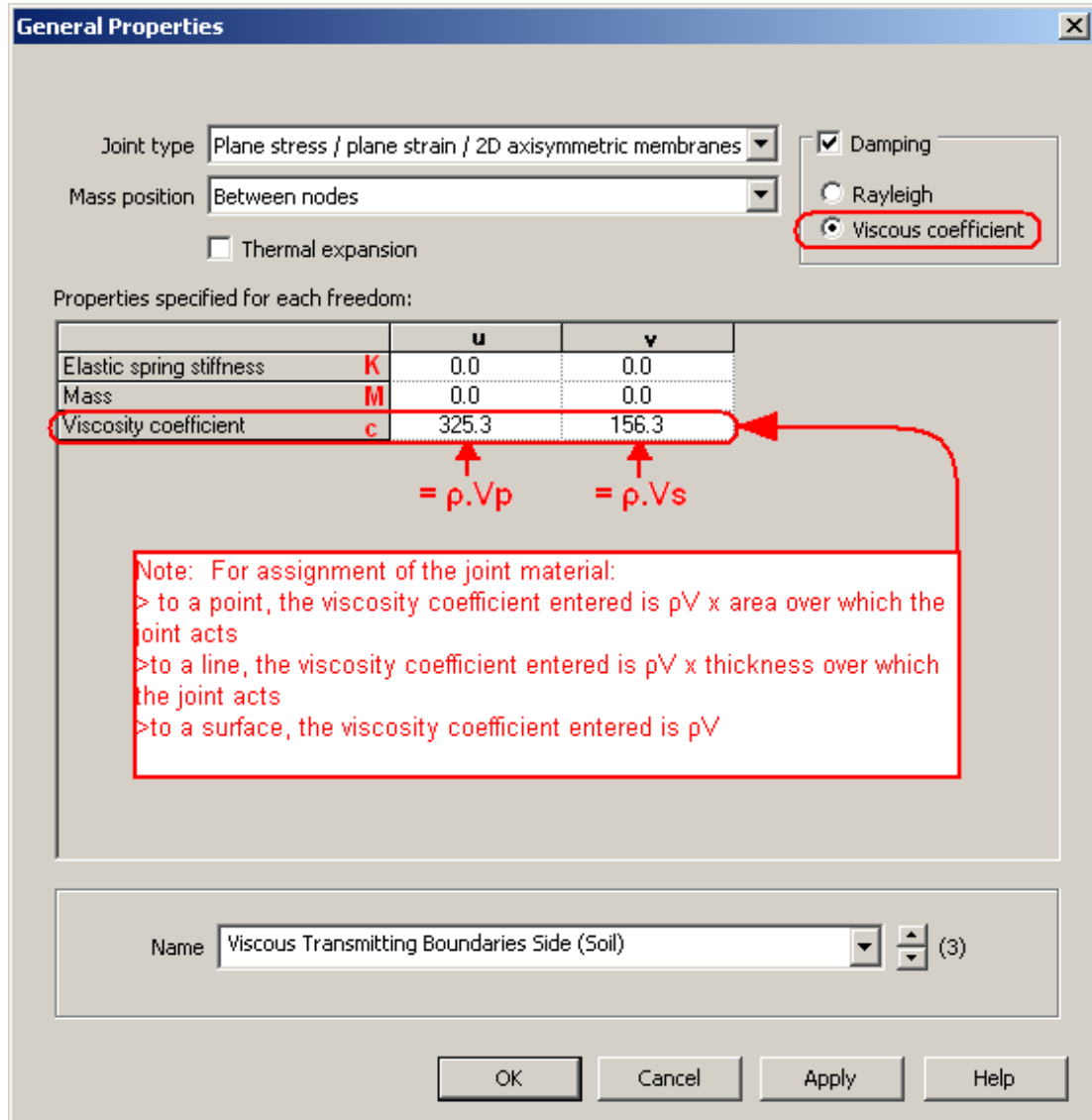


Figure 6 Joint Material - Entering Viscous Damping Coefficient

3.1 Example of Joint Properties

The following example shows the calculation for a 2D Plane Strain model. Homogenous linear elastic isotropic materials are assumed for this example's calculations.

3.1.1 Soil - Shear Direction

Soil material properties:

$$E = 34.7E3 \text{ kN/m}^2, \mu = 0.35, \rho = 1.9 \text{ t/m}^3$$

Therefore the impedances are calculated as follows:

$$\text{Shear Modulus, } G = E/(2*(1 + \mu)) = 12.85E3 \text{ kN/m}^2$$

$$\text{S-wave impedance, } Z_s = 1.9*V_s = 1.9*\sqrt{(G / \rho)} = 156.3 \text{ t/m}^2/\text{s}$$

Therefore in the shear direction:

$$c \text{ per unit area} = \beta = Z_s = 156.3 \text{ t/m}^2/\text{s}$$

3.1.2 Soil - Normal Direction

The normal stress induced by an elastic pressure wave (P-Wave) is given by:

$$\text{P-Wave Modulus, } M = E(1 - \mu)/((1 + \mu)(1 - 2\mu)) = 55.69E3 \text{ kN/m}^2$$

$$\text{P-wave impedance, } Z_p = 1.9*V_p = 1.9*\sqrt{(M / \rho)} = 325.3 \text{ t/m}^2/\text{s}$$

Therefore in the normal direction:

$$c \text{ per unit area} = \beta = Z_p = 325.3 \text{ t/m}^2/\text{s}$$

The joint properties as entered into the model for the soil boundaries (per unit length and for a plane strain mesh of unity thickness) are shown in Figure 7.

Note that for this example the joint element axes are assumed to be orientated to match the global axes and hence, column 1 relates to the global X direction and column 2 relates to the global Y direction.

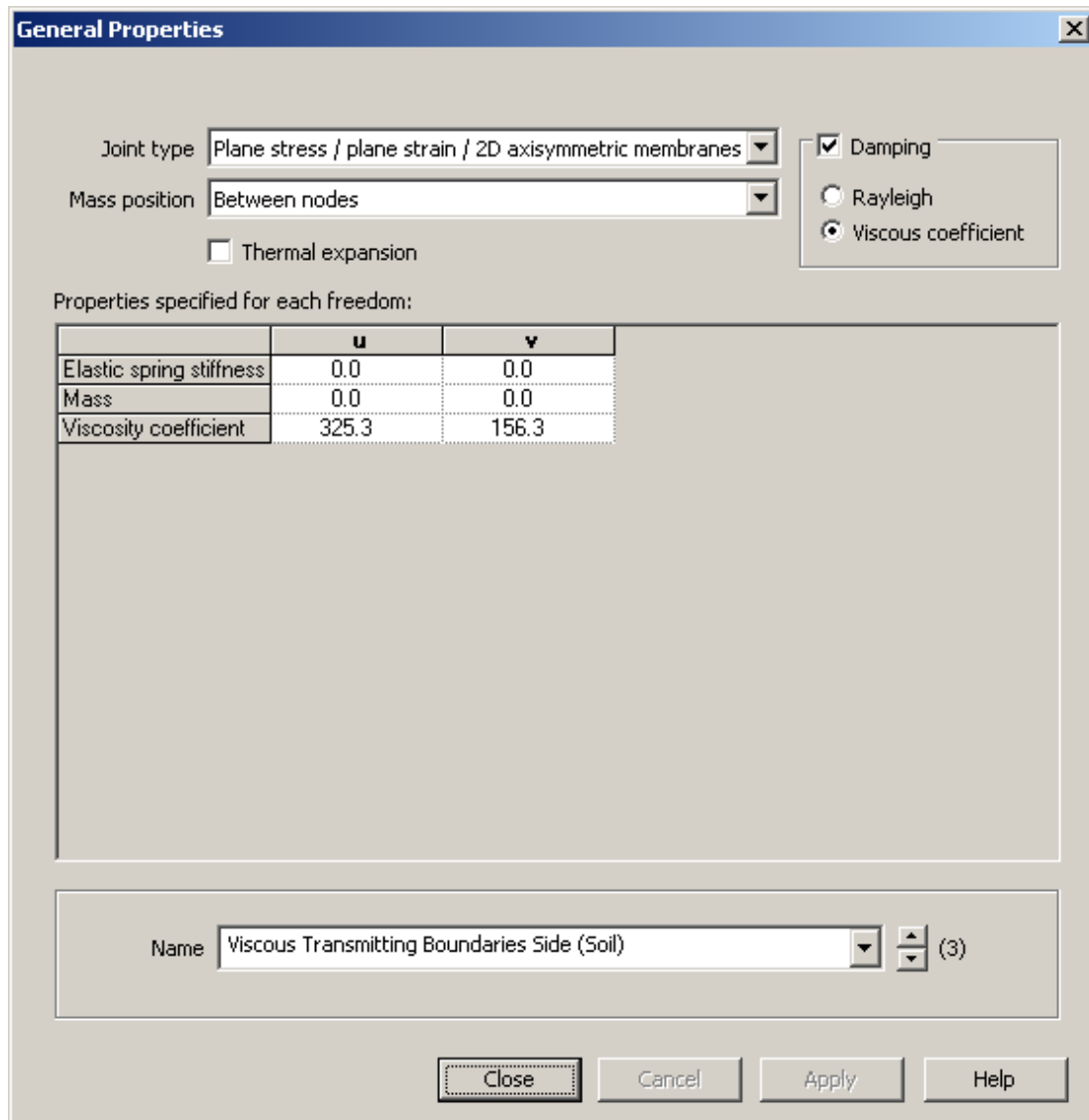


Figure 7 Example Joint "General Properties" for a Viscous Transmitting Boundary in a 2D Plane Strain Model – Soil Side Boundary

3.1.3 Rock - Shear Direction

Rock material properties:

$$E = 50E6 \text{ kN/m}^2, \mu = 0.3, \rho = 2.7 \text{ t/m}^3$$

Therefore the impedances are calculated as follows:

$$\text{Shear Modulus, } G = E / (2 * (1 + \mu)) = 19.23E6 \text{ kN/m}^2$$

$$\text{S-wave impedance, } Z_s = 2.7 * V_s = 2.7 * \sqrt{(G / \rho)} = 7.206E3 \text{ t/m}^2/\text{s}$$

Therefore in the shear direction:

$$c \text{ per unit area} = \beta = Z_s = 7.206E3 \text{ t/m}^2/\text{s}$$

$$\text{Stress due to an S-Wave, } \tau = 7.206E3 * v$$

3.1.4 Rock - Normal Direction

The normal stress induced by an elastic pressure wave (P-Wave) is given by:

$$\text{P-Wave Modulus, } M = E(1 - \mu) / ((1 + \mu)(1 - 2\mu)) = 67.31E3 \text{ kN/m}^2$$

$$\text{P-wave impedance, } Z_p = 2.7 * V_p = 2.7 * \sqrt{(M / \rho)} = 13.48E3 \text{ t/m}^2/\text{s}$$

Therefore in the normal direction:

$$c \text{ per unit area} = \beta = Z_p = 13.48E3 \text{ t/m}^2/\text{s}$$

$$\text{Stress due to a P-wave, } \sigma_p = 13.48E3 * v$$

The joint properties as entered into the model for the rock side boundaries are shown in Figure 8.

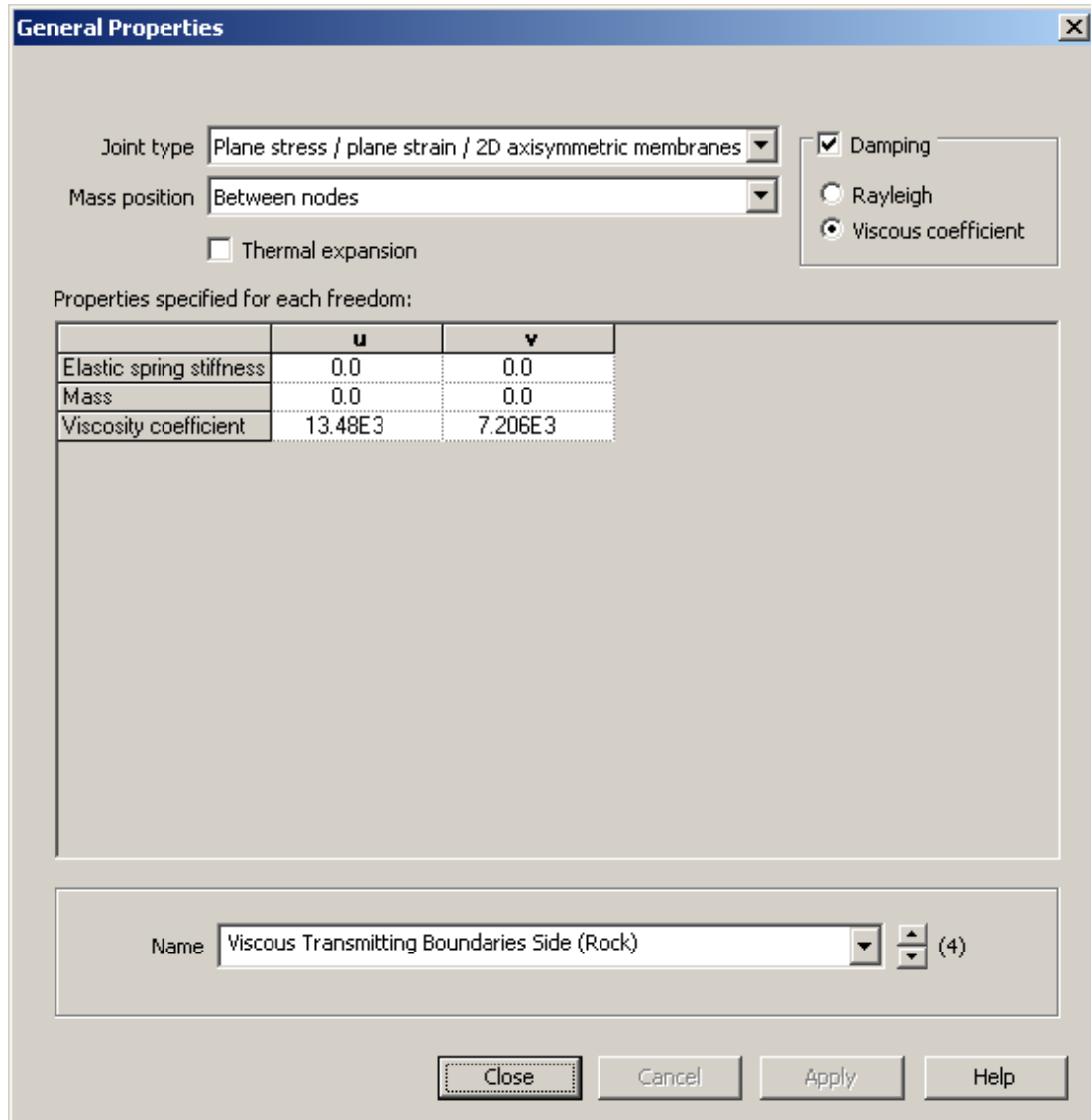


Figure 8 Example Joint "General Properties" for a Viscous Transmitting Boundary in a 2D Plane Strain Model – Rock Side Boundary

The joint properties as entered into the model for the rock base boundary are shown in Figure 9. The joint materials are assigned to the master assignment lines of the joint mesh at the respective boundaries and the material assignments for the example model are illustrated in Figure 10 .

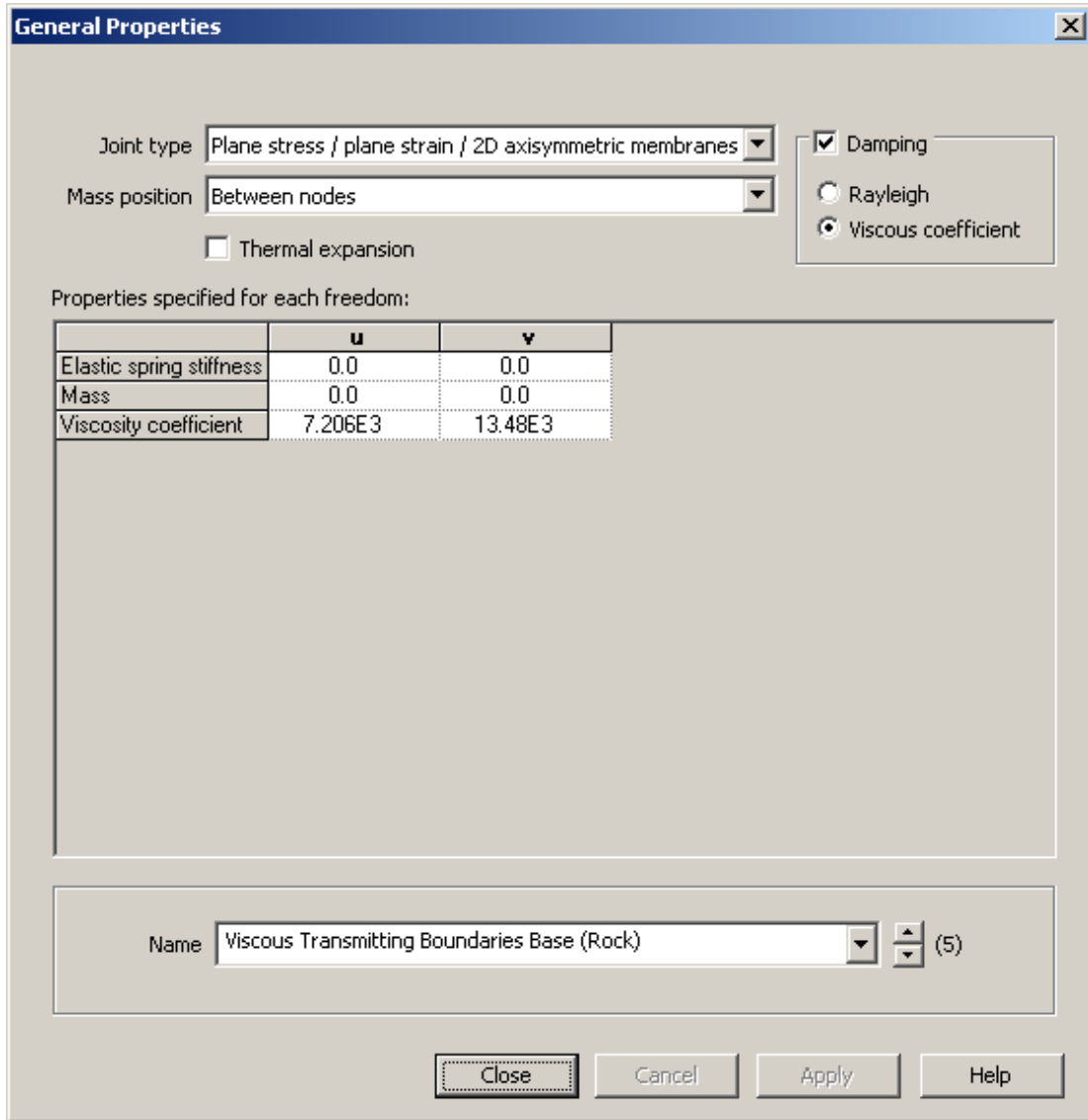


Figure 9 Example Joint "General Properties" for a Viscous Transmitting Boundary in a 2D Plane Strain Model – Rock Base Boundary

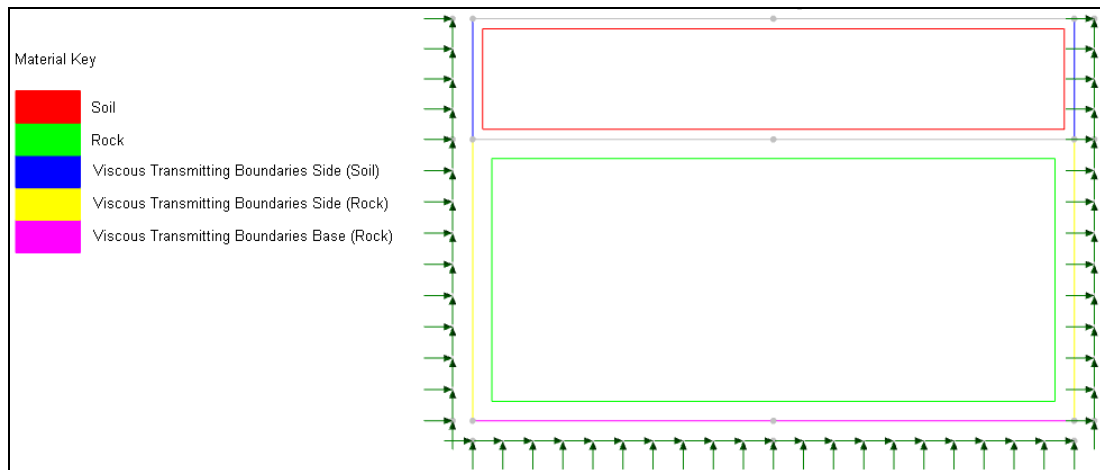


Figure 10 Example Joint "General Properties" for a Viscous Transmitting Boundary in a 2D Plane Strain Model – Material Assignments

3.2 Simple Test Model

The transient dynamic response of the model depicted in the images above was tested with a single sinusoidal cycle of vertical load applied at top dead centre as shown in Figure 11. This test load simulates an excitation source which is located at the surface level.

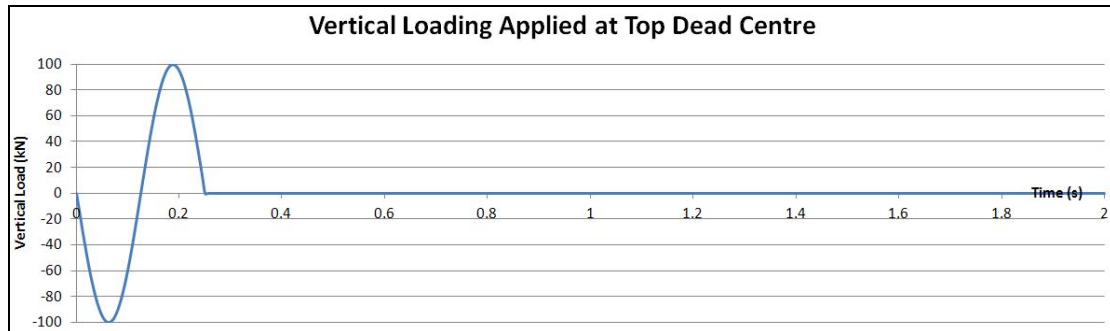


Figure 11 Single Sinusoidal Cycle of Vertical Load Applied to Top Dead Centre in a Simple Viscous Transmitting Boundary Test Model

This is a very simple test model just for the purposes of checking the transmitting boundary conditions are working as intended and assumes:

- Homogenous linear elastic isotropic materials
- No material damping in the meshed soil and rock
- No initial in-situ loads
- Completely drained ground.

Comparing the results with a larger model shows effective transmission of the incident waves after the waves have reached and passed the transmitting boundaries. A sample time step is shown in Figure 12 with effective strain contoured and exaggerated displacements.

The results of the model extended in the X direction are also very similar, confirming that there is little reflection of incident waves at the boundaries. As expected, some reflection of waves incident to the soil and rock interface is also evident, with the reflected waves apparent superimposed on the original propagating wave in the soil layer, and with some albeit diminished activity remaining near the epicentre.

The conventional approach with non-transmitting (reflecting) boundaries is also shown at the top of Figure 12, where the effect of the reflected waves and the closed system without any loss of energy is clear.

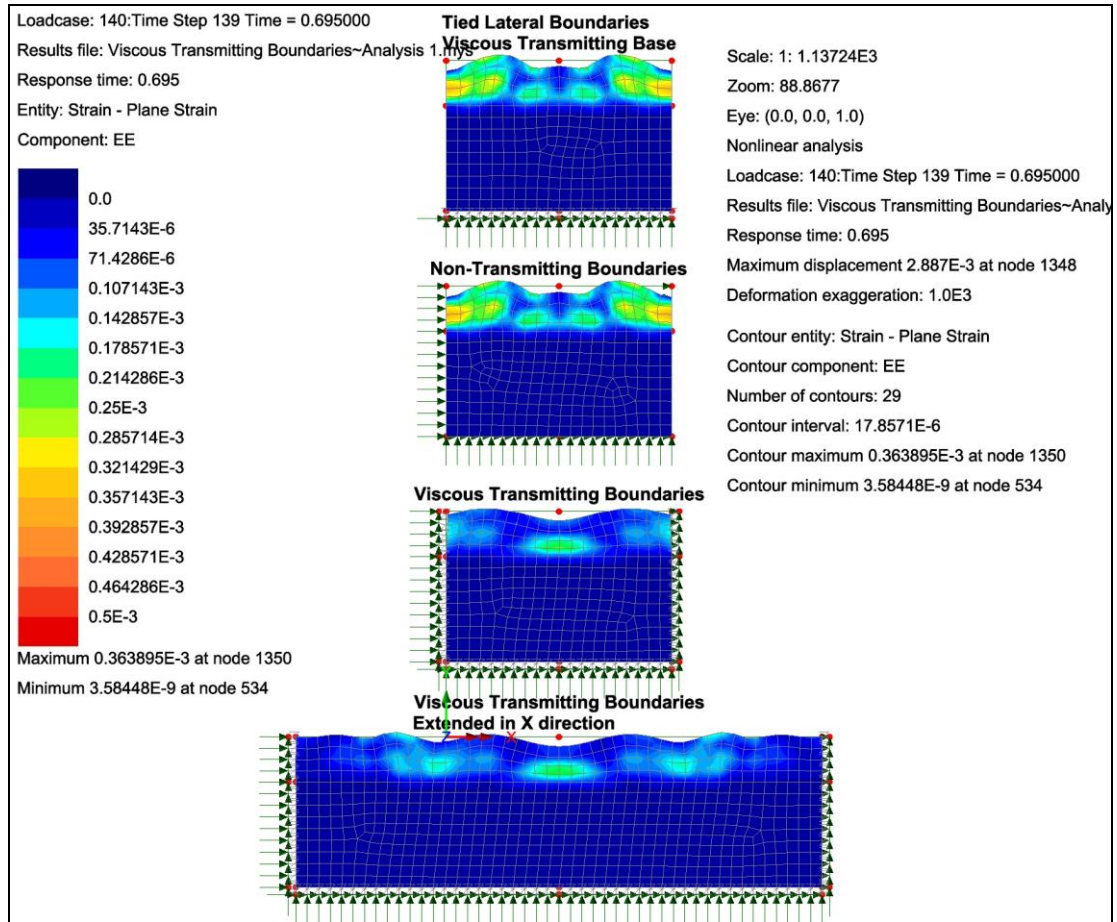


Figure 12 Comparison of the Response of a System with Conventional Non-Transmitting Boundaries (top) to one with Viscous Transmitting Boundaries (Middle) and one Extended in the X Direction (Bottom) due to a Single Sinusoidal Cycle of Vertical Load

4. Bicanic Boundaries

The conventional approach often adopted when performing a seismic analysis in a finite element analysis is to rigidly fix the base of the model and apply the earthquake as a prescribed acceleration to the fixed base as a 'support motion'. However to achieve reliable modelling a substantial depth of rock may need to be modelled at the base. This approach with a fixed non-transmitting boundary at the base inevitably leads to reflection of incident waves.

Transmitting boundary conditions at the base are required for the case where truncation of the model vertically at a convenient arbitrary depth is desirable, for example where there is no definite boundary between softer soil layers and bedrock, or if the bedrock is at an excessive depth. The truncation at this arbitrary level may not necessarily be in bedrock or a layer of linear homogeneous material and so transmission of the incident waves will be necessary. If the model can sensibly be truncated at a depth that is in bedrock, then Bicanic et al. suggest that practically no waves get transmitted back into the bedrock in such case and there is no need for transmitting boundary conditions [B1]. The reason given for this is that the transmitted wave is typically smaller than that reflected from the surface or interfaces between different soil layers because the bedrock is more rigid.

Reflected and incident waves cancel each other out to an extent at certain frequencies, making the earthquake input at an arbitrary level incorrect and a "deconvolution" procedure is therefore required to adjust the base input motion to account for the uneven frequency response of the truncated base. The traditional 1-D deconvolution approach based on a vertical slice of the model has serious shortcomings when applied to a true 2-D (or 3-D) model. The deconvolved input motion would only be correct for that vertical slice considered, and under linear elastic conditions. Elsewhere (under walls, for example) the base motion is incorrect, leading to trapped standing waves and over-conservative response predictions.

Where the conventional approach of applying a support motion is employed, fixed nodes are required to apply the earthquake input and so the use of a viscous transmitting boundary at the base of the model is not an option.

This problem was addressed by Professor Bicanic and colleagues at Swansea in the 1980's [B1]. Bicanic et al. proposed an alternative input motion definition, based on an analysis of motion relative to the incoming seismic wave. A viscous transmitting boundary is included at the base of the model (though it is noted that other idealisations of transmitting boundaries can be used instead if desired [B1]), and loads are applied as boundary tractions (Face Loads) proportional to velocity and internal body forces (CBF's) proportional to acceleration. The method eliminates the need for deconvolution altogether, at the expense of some additional loading set-up and results processing effort. Solutions obtained with this boundary condition are essentially independent of model depth, even for relatively shallow rock meshes.

The Bicanic boundary condition (BBC) can be implemented in LUSAS using joint elements configured as dashpots, together with pressure and acceleration loads. This approach, described herein, considers a vertically propagating plane wave (compression and shear) and assumes that the incoming waves applied at the base do not vary laterally along the base.

4.1 Lateral Boundaries

For the method proposed by Bicanic et al., the lateral boundaries are required to transmit only outgoing waves caused by the structure modelled or surface perturbation. Therefore the lateral transmitting boundaries are set up to transmit the difference between the main mesh displacement and the free-field displacement.

Bicanic et al. [B1] proposed the use of “tied” lateral boundary conditions where constraint equations are applied to pairs of nodes on opposite lateral boundaries to keep their degrees of freedom the same. Thus in effect modelling a continuous series of models repeated infinitely laterally from each boundary pitched at the width of the model. This approach was compared with the approach using viscous transmitting boundaries and found to give similar results assuming that the width/pitch (or spacing between structures) is sufficiently large and thus Bicanic suggests this approach is used as standard because it facilitates the modelling of full nonlinear behaviour. The only area of the model where linear material behaviour must be maintained with this approach is at the base of the model where viscous transmitting boundaries are still required.

For the lateral boundaries in the LUSAS model, “tied” lateral boundary conditions are achieved by using a **Tied Mesh** constraint equation (**Attributes > Constraint Equation > Tied Mesh . . . > Normal Constraint**) which is assigned to the model as shown in Figure 13. The same model as used to demonstrate viscous transmitting boundaries is used here to illustrate the development of the model to apply Bicanic boundary conditions.

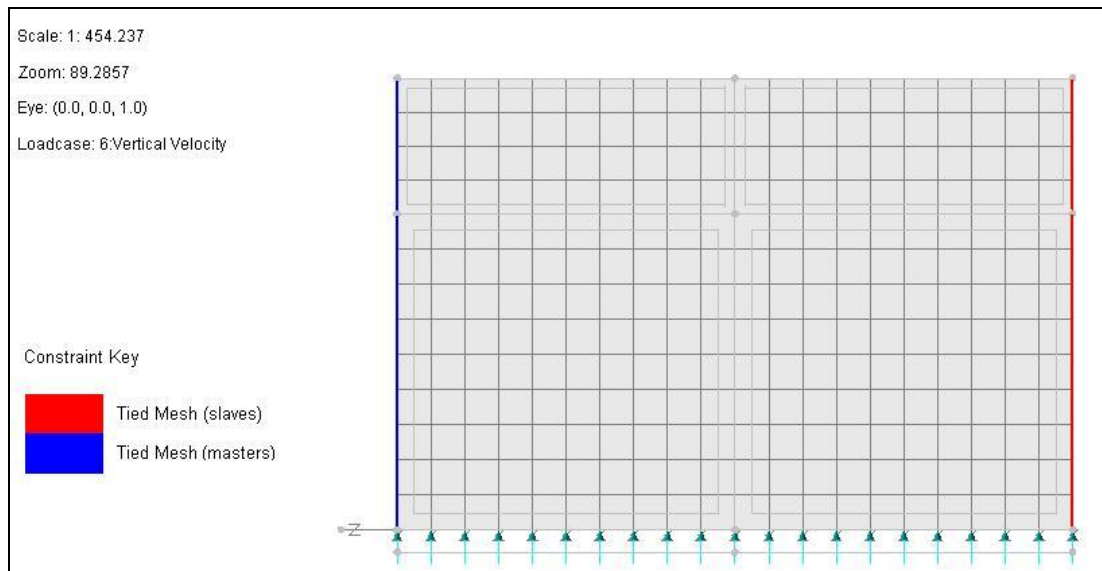


Figure 13 Tied Mesh Assignment

Note that a **Cyclic Translation** constraint equation (**Attributes > Constraint Equation > Cyclic . . . > Translational Constraint**) could be used instead of a **Tied Mesh** constraint if preferred to achieve the same set of constraint equations between pairs of nodes on these opposite lateral boundaries.

If viscous transmitting boundaries are used for the lateral boundary conditions the approach is more involved requiring a 1D free-field model to be run in parallel to tune the lateral dampers to transmit only the outgoing waves due to the presence of the structure or other modelled perturbation. Bicanic et al. [B1] explains this procedure further in section 4.2 "Transmitting" layer approach.

Where joint material properties need to vary with depth, a variation (**Utilities>Variation**) can be used to vary the viscous damping coefficients assumed with the depth at any location on a vertical boundary. It should be noted that this approach does assume linear material behaviour in the region at the lateral truncation boundaries.

4.2 Joints and Material Assignments

The vertical and horizontal impedance of the base rock is first calculated from the respective wave speeds. This impedance is used to factor the surface pressures at the base of the model, as well as to define the dashpot constants.

In the same way as used for the viscous transmitting boundaries, impedances and joint material properties for the base can be calculated.

4.2.1 Rock - Shear Direction

Rock material properties:

$$E = 50E6 \text{ kN/m}^2, \mu = 0.3, \rho = 2.7 \text{ t/m}^3$$

Therefore the impedances are calculated as follows:

$$\text{Shear Modulus, } G = E / (2 * (1 + \mu)) = 19.23E6 \text{ kN/m}^2$$

$$\text{S-wave impedance, } Z_s = 2.7 * V_s = 2.7 * \sqrt{(G / \rho)} = 7.206E3 \text{ t/m}^2/\text{s}$$

Therefore in the shear direction:

$$c \text{ per unit area} = \beta = Z_s = 7.206E3 \text{ t/m}^2/\text{s}$$

$$\text{Stress due to an S-Wave, } \tau = 7.206E3 * v$$

4.2.2 Rock - Normal Direction

The normal stress induced by an elastic pressure wave (P-Wave) is given by:

$$\text{P-Wave Modulus, } M = E(1 - \mu) / ((1 + \mu)(1 - 2\mu)) = 67.31E3 \text{ kN/m}^2$$

$$\text{P-wave impedance, } Z_p = 2.7 * V_p = 2.7 * \sqrt{(M / \rho)} = 13.48E3 \text{ t/m}^2/\text{s}$$

Therefore in the normal direction:

$$c \text{ per unit area} = \beta = Z_p = 13.48E3 \text{ t/m}^2/\text{s}$$

$$\text{Stress due to a P-wave, } \sigma_p = 13.48E3 * v$$

The joint properties as entered into the model for the rock base boundary are shown in Figure 14. The joint materials are assigned to the master assignment lines of the joint mesh at the base of the model only

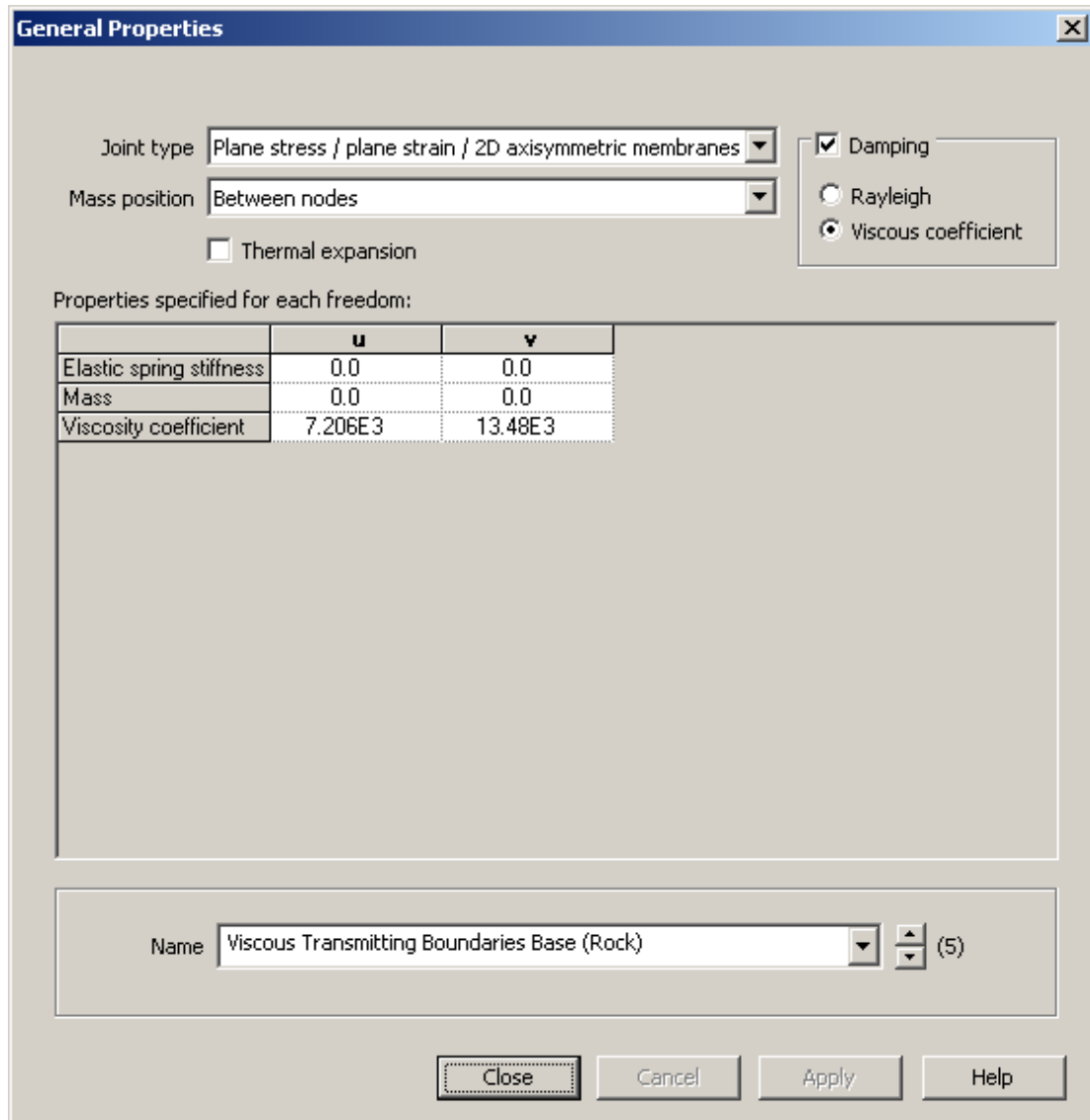


Figure 14 Example Joint "General Properties" for the Bicanic Boundary Conditions for the Base of a 2D Plane Strain Model – Rock Base Boundary

A single 'dummy' joint is also required that will have all material properties set to zero. The assignment of material properties is shown in Figure 15 for a 2D Plane Strain model. This dummy joint connects between a node on the continuum mesh and a separate single 'dummy' node as shown in Figure 16. Whilst the 'dummy' node is supported in X and Y, this node will be applied equal and opposite acceleration to the continuum as explained further in 4.3.

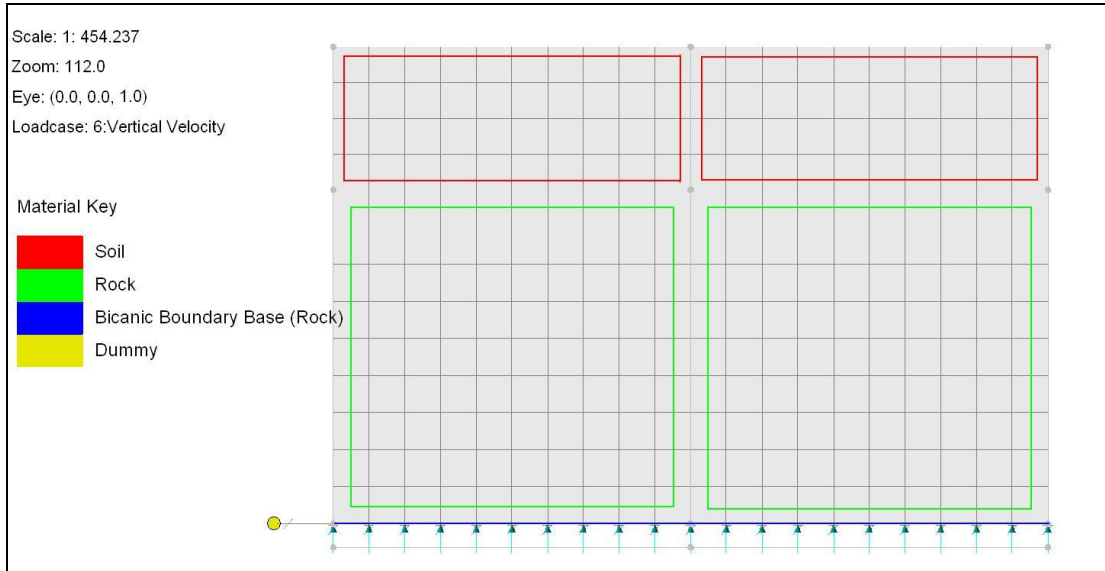


Figure 15 Material Assignments

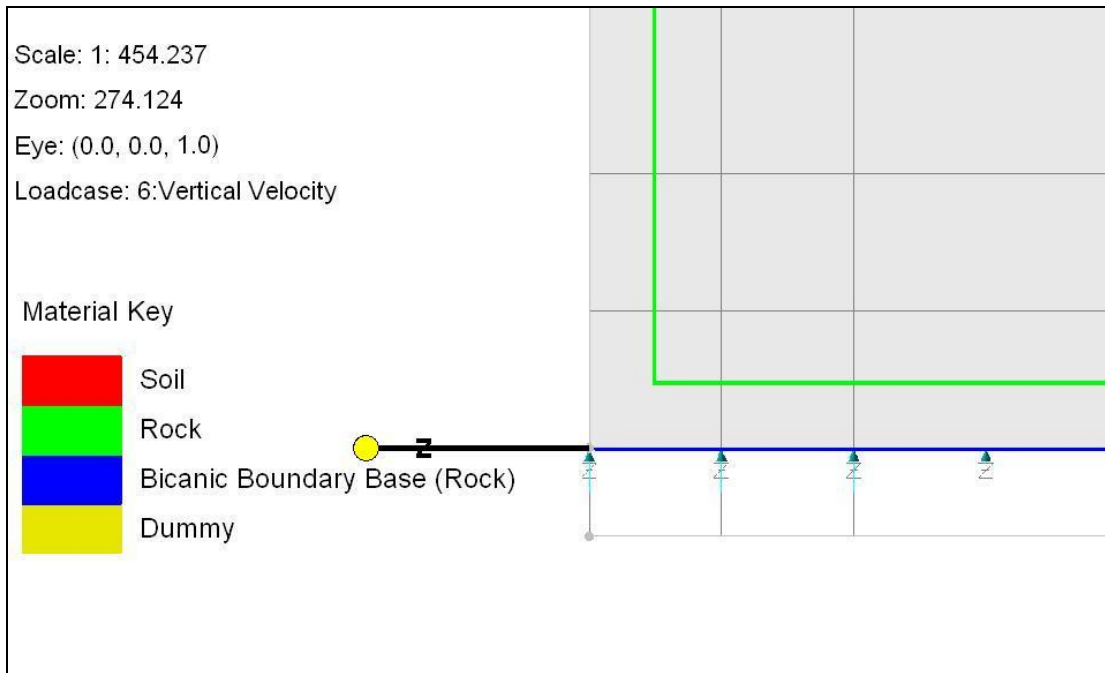


Figure 16 Dummy Joint (Shown Highlighted in Black)

4.3 Load Curves and Loading Assignments

The hard site acceleration history must also be integrated to obtain velocity. There are software applications available that can perform this integration, or the integration could be calculated in a spreadsheet. Both acceleration and velocity time histories are then put in to the LUSAS model as load curves.

The hard site acceleration time histories are entered as load curves (**Utilities>Load Curve**). In order to input the full seismic load, half the hard site acceleration is applied as a CBF load to all elements in one direction, and half the hard site velocity is applied to the pressure loads on the base of the model in the opposite direction. This is shown schematically for the input of the vertical motion in Figure 17

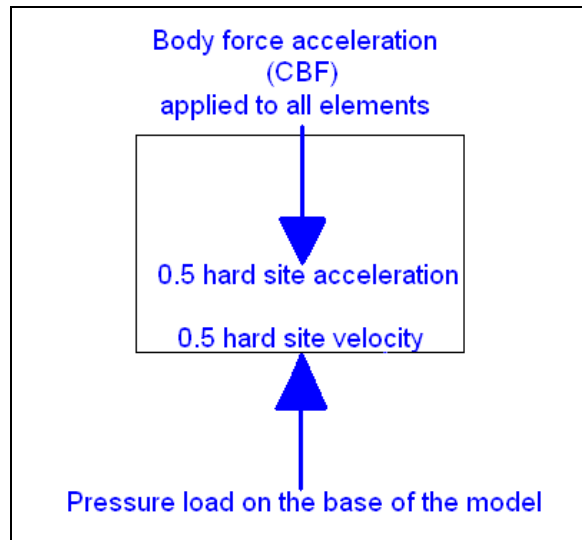


Figure 17 Input of the Vertical Seismic Load

The resulting response will be motion relative to the incoming wave (i.e. 0.5 x hard site motion). Accordingly, (0.5 x hard site motion) must be added to all responses to recover total motion. This is achieved by defining a dummy node in the model with a prescribed acceleration load of (0.5 x hard site motion) in the opposite direction to the CBF. This simplifies the correction calculation of the total response because LUSAS Solver will then do the integration of the accelerations at the 'dummy' node to get the velocity and again to get displacement, all naturally in the course of running the solution, and for the negligible computational cost of just one additional node and element in the model.

Four basic load curves are therefore defined:

1. 0.5 m/s² Horizontal Acceleration applied to all elements as a negative acceleration (CBF)
2. 0.5 m/s² Vertical Acceleration applied to all elements as a negative acceleration (CBF)
3. (0.5 x Density x Vs) Shear pressure on base of model
4. (0.5 x Density x Vp) Normal pressure on base of model

Where: V_s = Shear Wave Speed
 V_p = Pressure Wave Speed

The assignment of the loads to each load curve as described is shown in Figure 18.

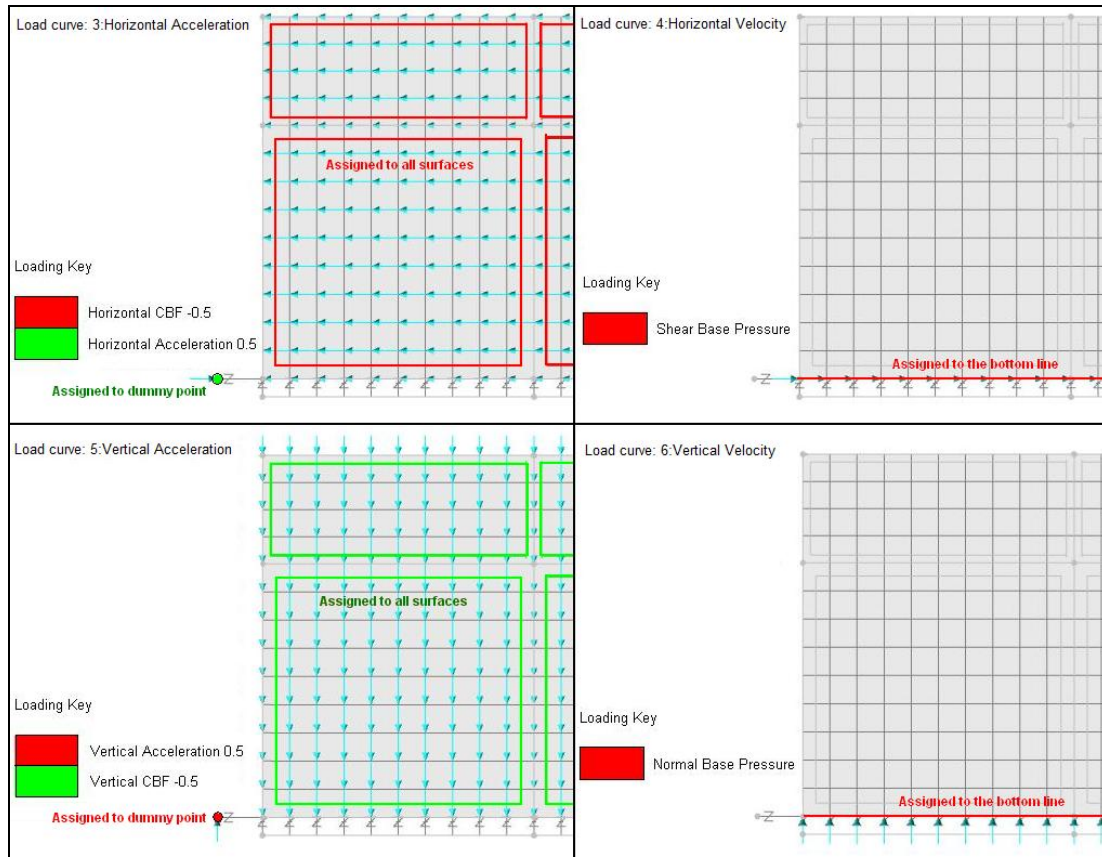


Figure 18 Load Assignments to Each Load Curve

The load curves and loadcase set up in LUSAS Modeller are shown in Figure 19 with the four load curves as defined above, but only one loadcase is needed. The pressure loads are assigned to their respective load curves with the load factor set equal to the value of the impedance calculated for that direction (density x wave speed). The loads assigned to the vertical load curves show a factor of 2/3 also applied. The raw earthquake data has been input into the load curve but in this case a factor of 2/3 for the vertical direction has been applied to the vertical acceleration body force downwards, and also to the equal and opposite normal base pressure and dummy node vertical acceleration to achieve the required excitation for this particular analysis. The design code you are working to should be referred to for the factors that need to be considered.

A single loadcase is used with the supports assigned to it and a **Nonlinear and Transient** control set up to run an implicit dynamics analysis in the time domain. Note that the particular settings in the screenshot just assume a very simple setup; other loadcases may be required and the Nonlinear option may need to be turned on.

Note:
 The "Shear Base Pressure" and "Normal Base Pressure" loads are 0.5. The load factor that they are assigned with therefore makes them equal and opposite to the body force accelerations. The factors are therefore the impedances calculated for that direction. Hence "7.2063E3" for the "Shear Base Pressure" and "8.98667E3" (=2/3 x 13.48E3) for the "Normal Base Pressure"

	Time	Factor
1	0.0	0.0
2	5.0E-3	-0.0107
3	0.01	-0.0157
4	0.015	-0.0190
5	0.02	-0.0259
6	0.025	-0.0286
7	0.03	-0.0357

Figure 19 Bicanic Boundaries - Loadcase Treeview Load Curves and Transient Control

5. Other Considerations

For a seismic analysis of the type considered in this note there will also be a number of other advanced considerations, but whilst some have been briefly mentioned, they are not fully covered in this note. Some of these other considerations include dynamic soil stiffness under seismic loads, kinematic hardening of soil, initial stress conditions, the modelling of the water level and drained versus un-drained conditions for example.

6. References

- B1** Zienkiewicz, Bicanic, Shen "*Earthquake Input Definition and the Transmitting Boundary Conditions*", Department of Civil Engineering, University College, Swansea, UK, 1986
- L1** LUSAS "*Modeller Reference Manual*", LUSAS Version 14.7 : Issue 1, LUSAS, UK, 2011
- L2** LUSAS "*Theory Manual Volume 1*", LUSAS Version 14.7 : Issue 1, LUSAS, UK, 2011
- W1** Wolf, J.P. *Soil-Structure Interaction Analysis in Time Domain*. Prentice-Hall.