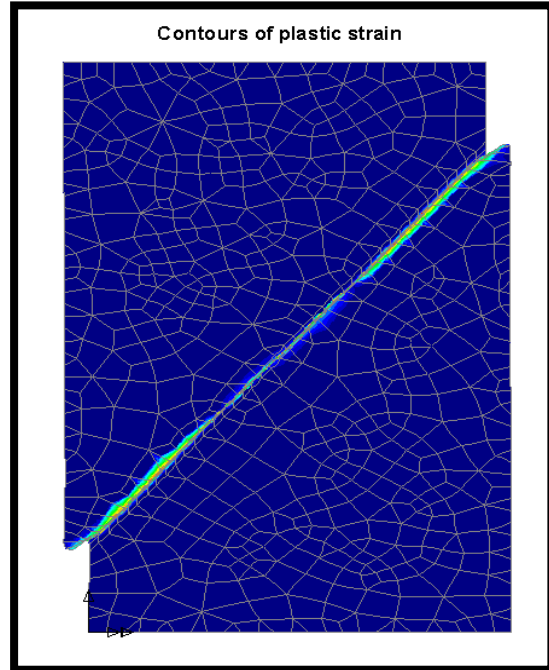


1. Plane strain tests on London clay

1.1. Drained simulation

Plane strain drained simulations have been carried out on a 200 x 100 mm sample of London clay. The first loadcase consists of the application of an isotropic initial stress field around the complete sample before loading in the vertical direction takes place in subsequent loadcases. The initial applied confining stress is applied directly to the gauss points of the elements and does not invoke any straining in the sample; it is purely to set the initial starting condition before loading in the vertical direction takes place.

Compression of the soil takes place via the application of a vertical downward displacement over the complete width of the sample and is controlled by the use of automatic non-linear incrementation. The sample was subjected to strains of up to 6.5% to ensure that yielding along a pre-determined shear plane had taken place.



The base of the sample is assumed to be fixed in the vertical direction and one corner of the base is fixed both horizontally and vertically.

A non-linear soil model using the Mohr-Coulomb failure criterion (Model No. 65) is used to represent the non-linear response of the London clay. This is a fairly crude, elastic-perfectly plastic non-linear model but is adequate for the purposes of these tests.

As this was a drained test, excess pore pressures were not accounted for and so this represented a test in which the load was applied very slowly.

Three such tests were carried out with different confining stresses of 100, 200 and 300 N/mm² respectively.

Soil properties in terms of effective stress used in the tests were as follows:

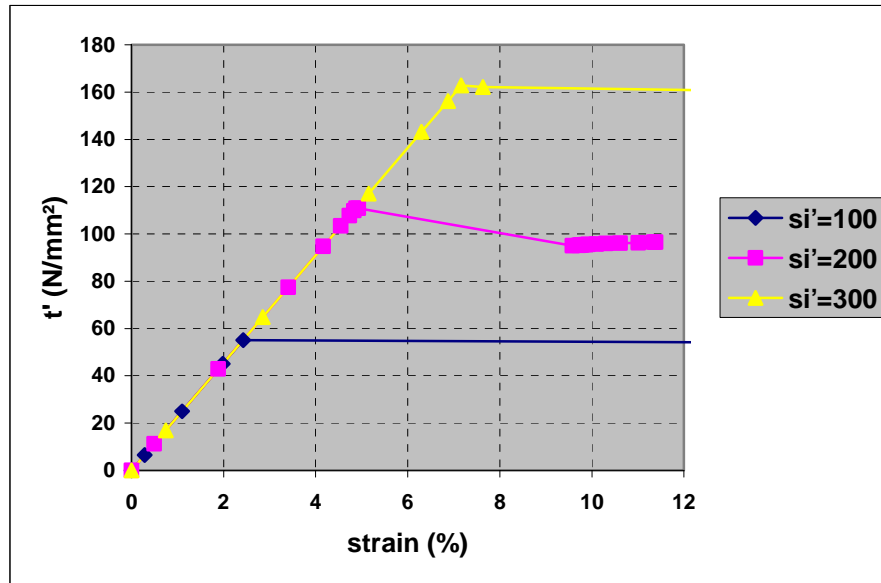
Young's modulus	E'	5000	N/mm ²
Poisson's ratio	ν'	0.2	
Cohesion	c'	5	N/mm ²
Angle of friction	ϕ'	20	degrees
Angle of dilation	ψ	0	degrees

The natural failure plane through the sample occurs at an angle of $(45 + \phi/2)^\circ$ to the horizontal and a 5mm thin band of very slightly weaker material was placed through the

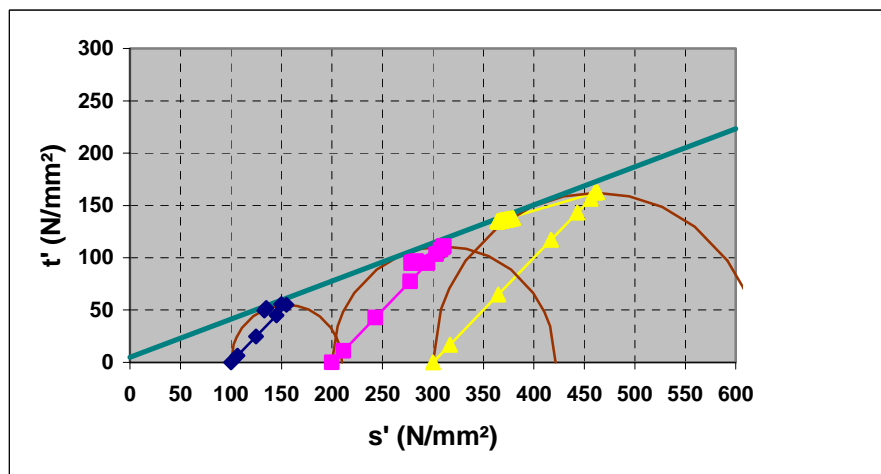
sample at this angle to ensure that the correct shear deformations occurred during loading.

[Refer to command files *drained_100.cmd*, *drained_200.cmd* & *drained_300.cmd*]

Results of effective shear stress against equivalent strain and effective stress path plots (normal stress against shear stress) for the three tests can be seen in the spreadsheet *plane_strain.xls* along with the resulting failure envelope.



Effective shear stress against equivalent strain

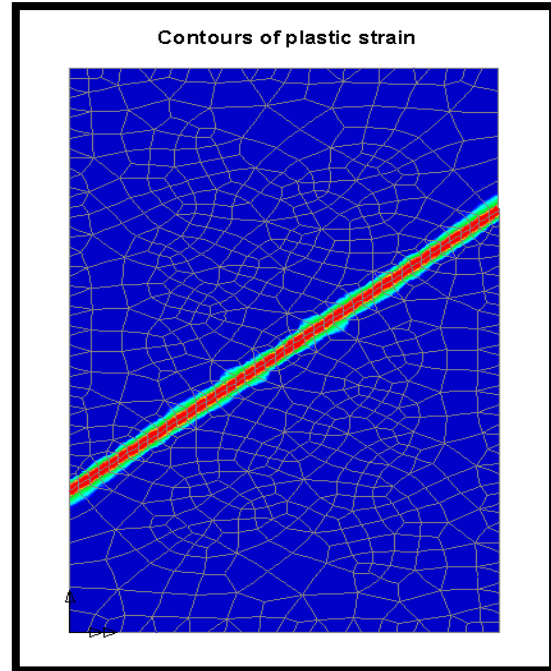


Effective stress paths

1.2. **Unconsolidated undrained simulation**

Undrained tests in terms of total stress were carried out on three samples of the same dimensions as the drained tests. The same support conditions and loads were also used but the soil parameters were now in terms of total stress rather than effective stress. The Mohr-Coulomb failure criterion was again used but as the friction angle used was zero, a Tresca failure criterion could have been used and similar results obtained. Although this was an undrained analysis, excess pore pressures were not calculated as standard plane strain elements have been used without pore pressure freedoms.

The initial confining stress was applied as a face load in the first load case rather than an initial stress so as to avoid consolidation of the sample.



Soil properties in terms of total stress were as follows:

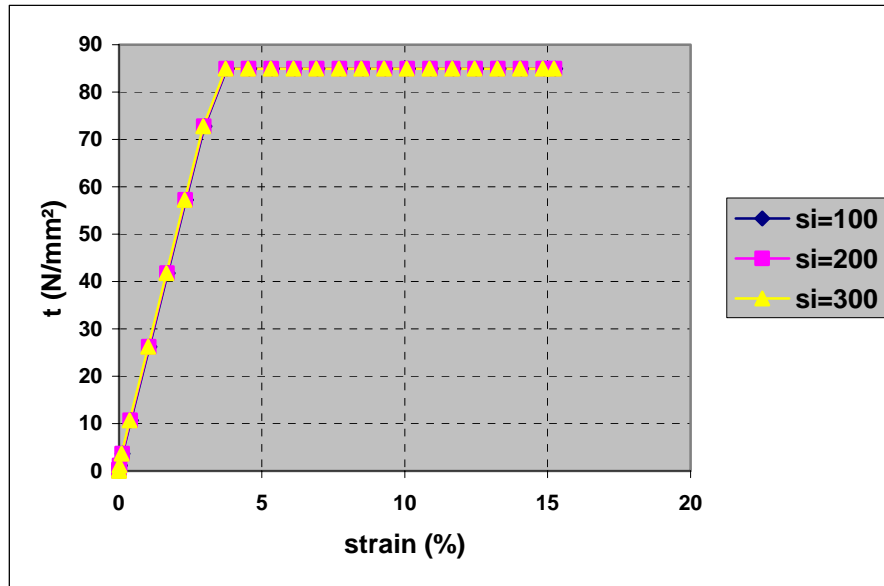
Young's modulus	E_u	6250	N/mm ²
Poisson's ratio	ν_u	0.495	
Undrained shear strength	C_u	85	N/mm ²
Friction angle	ϕ_u	0	degrees

The band of slightly weaker material was placed at an angle of 45° through the soil sample.

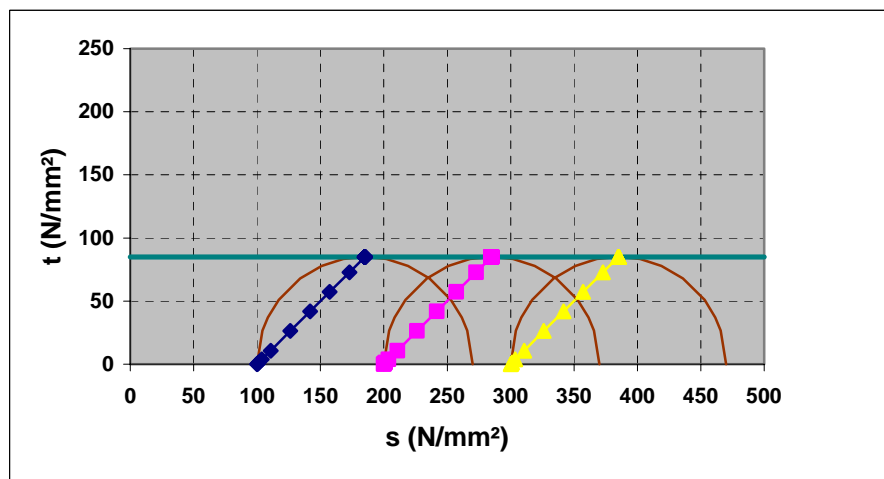
The results of the three simulations show that the sample deforms without changing in volume and that the yield stress is independent of the initial confining stress. This is because the sample is undrained and any changes in total stress are accompanied by an equivalent change in pore pressure giving rise to no change in effective stress.

[Refer to command files *undrained2_100.cmd*, *undrained2_200.cmd* & *undrained2_300.cmd*]

Results for these tests are also provided in the spreadsheet *plane_strain.xls*.



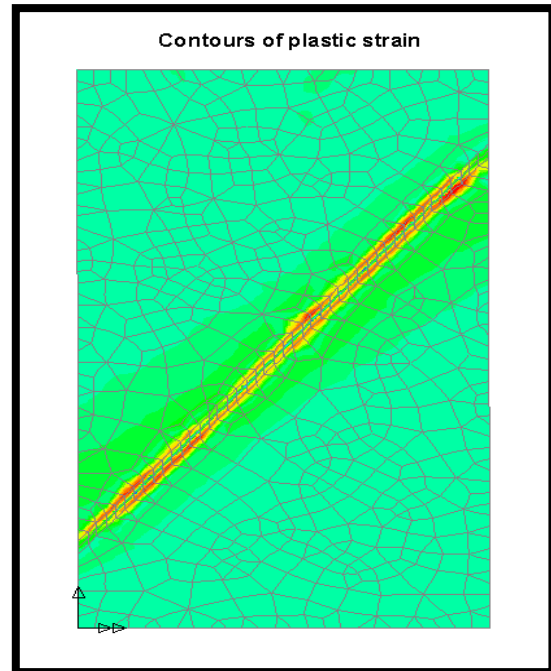
Total shear stress against equivalent strain



Total stress paths

1.3. Consolidated undrained simulation

Undrained tests in terms of effective stress were also carried out on samples of dimensions as before. Initial stresses were applied directly to the sample (thereby allowing consolidation) but then the displacement was applied under undrained conditions. Elements with pore pressure freedoms as well as displacement freedoms were used allowing the generation of excess pore pressure to be calculated. Identical strength and stiffness soil parameters, in terms of effective stress, were used as for the drained test however additional “two-phase” material properties were necessary for use with this element type. In this example the bulk modulus of the solid particles and the water were set to be the same value thereby removing the effects of porosity and setting the equivalent bulk modulus to be equal to the bulk modulus of water.

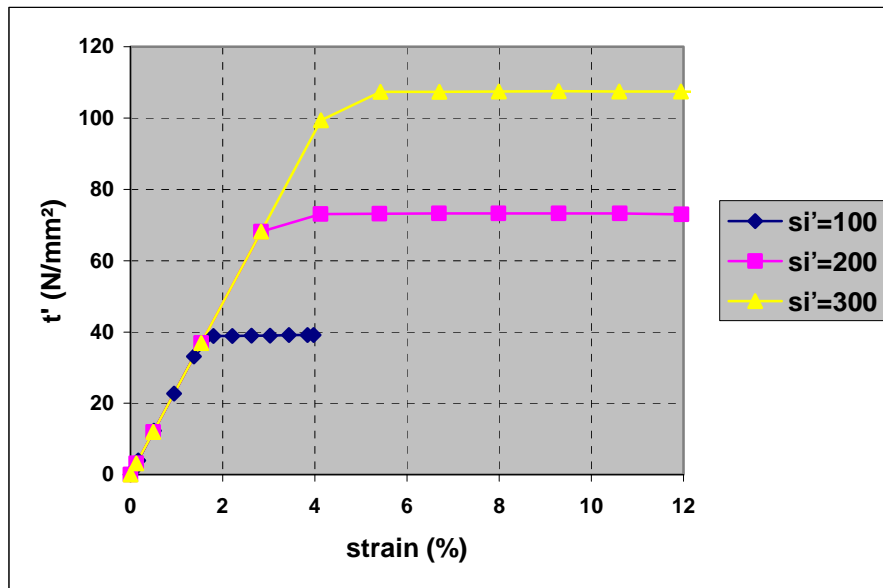


Bulk modulus of solid particles	K_s	2.1E+06	kN/m ²
Bulk modulus of water	K_w	2.1E+06	kN/m ²
Porosity	n	0.63	
Unit weight of fluid phase	γ_w	10	kN/m ³
Permeability in x direction	k_x	Not used	m/s
Permeability in y direction	k_y	Not used	m/s
Permeability in z direction	k_z	Not used	m/s

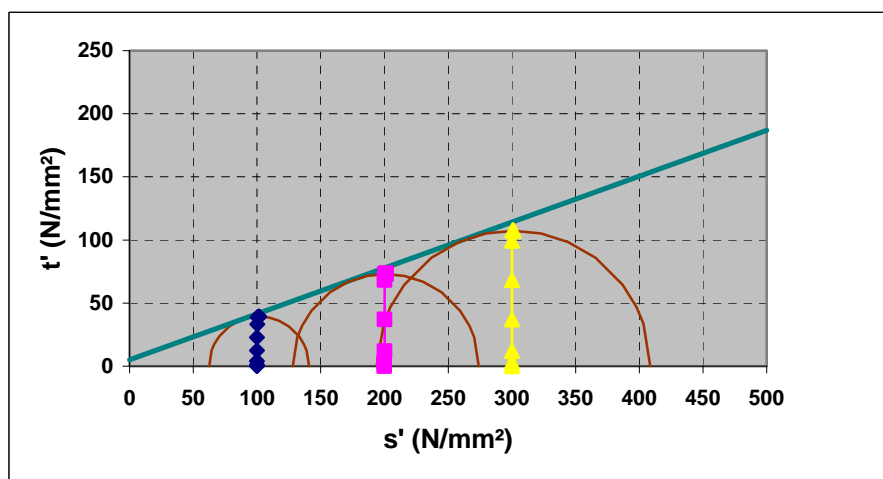
The results of these tests are very similar to the drained test results although the shear strength exhibit is slightly less. This is as expected as the loading occurs undrained, thereby inducing excess pore pressures which leads to a reduction in effective stress and shear strength. Note that the effective stress paths are vertical in this test as expected since there is no change in effective stress during the loading stage.

[Refer to command files *c_undrained_100.cmd*, *c_undrained_100.cmd* & *c_undrained_100.cmd*]

Results for these tests are included in the spreadsheet *plane_strain.xls*.



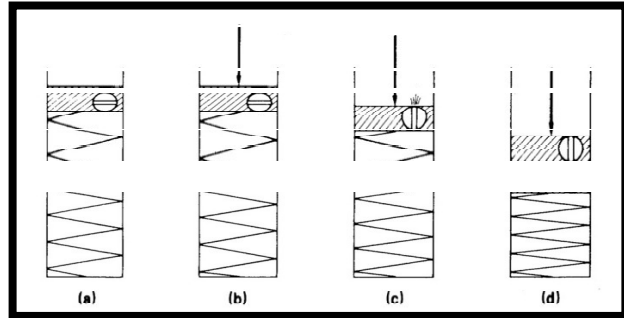
Effective shear stress against equivalent strain



Effective stress paths

2. One-dimensional consolidation

An analysis has been carried out to simulate an example of one-dimensional Terzaghi consolidation. A 10m high, saturated column of soil is subjected to a load of 10kPa in the first load increment. This generates a uniform excess pore pressure of 10kPa throughout the column. The base of the column is completely restrained.



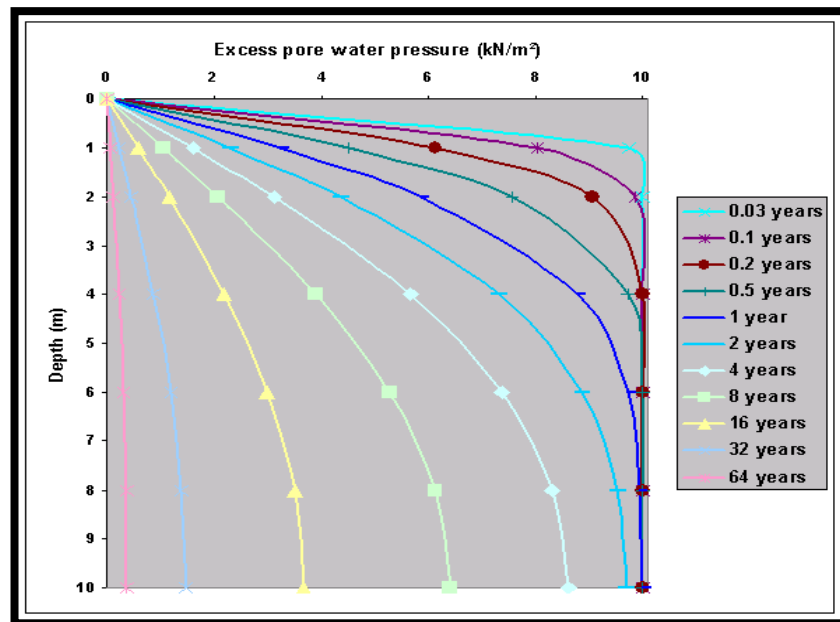
Drainage is allowed through the top surface only and as time progresses, the excess pore pressures reduce and settlement of the column takes place.

Soil parameters used in the analysis are as follows:

Young's modulus	E'	1000	kN/m^2
Poisson's ratio	ν'	0.25	
Bulk modulus of solid particles	K_s	$2.1\text{E}+06$	kN/m^2
Bulk modulus of water	K_w	$2.1\text{E}+06$	kN/m^2
Porosity	n	0.63	
Unit weight of fluid phase	γ_w	10	kN/m^3
Permeability in x direction	k_x	$1\text{E}-09$	m/s
Permeability in y direction	k_y	$1\text{E}-09$	m/s
Permeability in z direction	k_z	$1\text{E}-09$	m/s

Pore pressure isochrones showing the reduction in excess pore pressure with time through the height of the soil column is shown below.

[Refer to command files *one_d_consolidation.cmd*]



Pore pressure isochrones from 1-D Terzaghi consolidation

3. Slope stability analysis

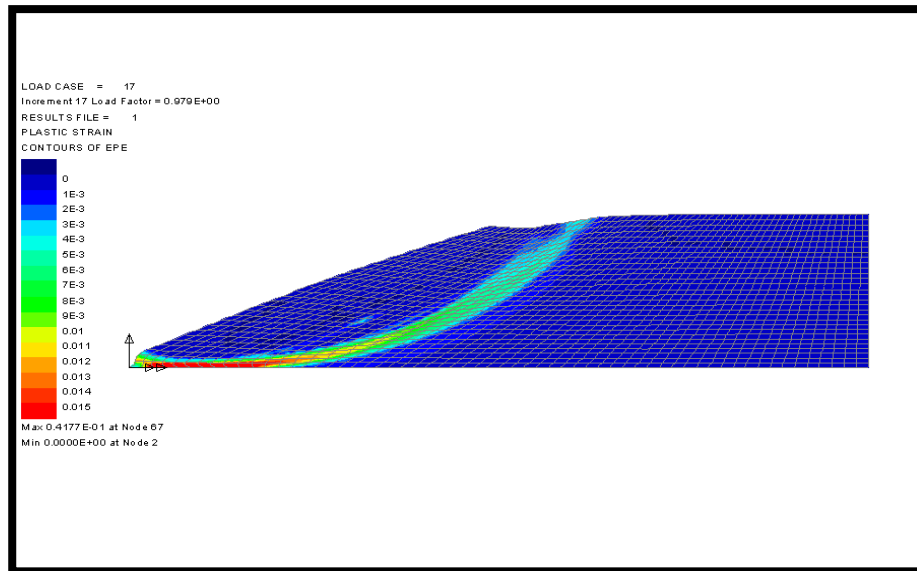
A weak 1:2 soil slope under a gravitational load has been analysed to determine the position of the naturally occurring slip surface profile. The soil strength parameters chosen were such as to cause the slope to fail and a load factor of 1.0 could not be achieved. The analysis fails with a “Current increment has failed to converge error”, achieving a load factor of 0.979.

Standard plane strain elements were used, ignoring pore pressure and the base of the model was assumed to be fully fixed.

Soil parameters used in the analysis were as follows:

Young's modulus	E'	100E3	kN/m^2
Poisson's ratio	ν'	0.3	
Density	ρ	2	kg/m^3
Cohesion	c'	10	kN/m^2
Angle of friction	ϕ'	11	degrees
Angle of dilation	ψ	0	degrees

[Refer to command file *slope.cmd*]



Contours of plastic strain to 1.5%

4. Footing analysis

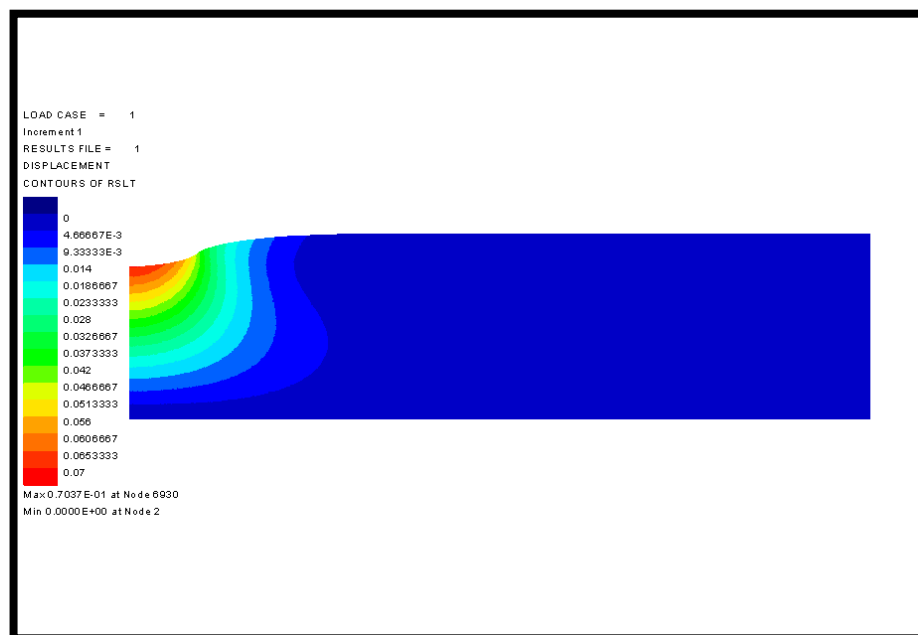
4.1. Drained analysis

An elastic plane strain analysis has been carried out to model the response of a soil continuum to a 4m wide uniformly distributed load of 30 kN/m². The analysis was carried out drained and therefore effective stress soil parameters were used.

Young's modulus	E'	3000	kN/m ²
Poisson's ratio	ν'	0.25	

A maximum vertical displacement of 70mm was predicted below the centre-line of the footing.

[Refer to command file *drained_footing.cmd*]



Displacement contours for drained footing analysis

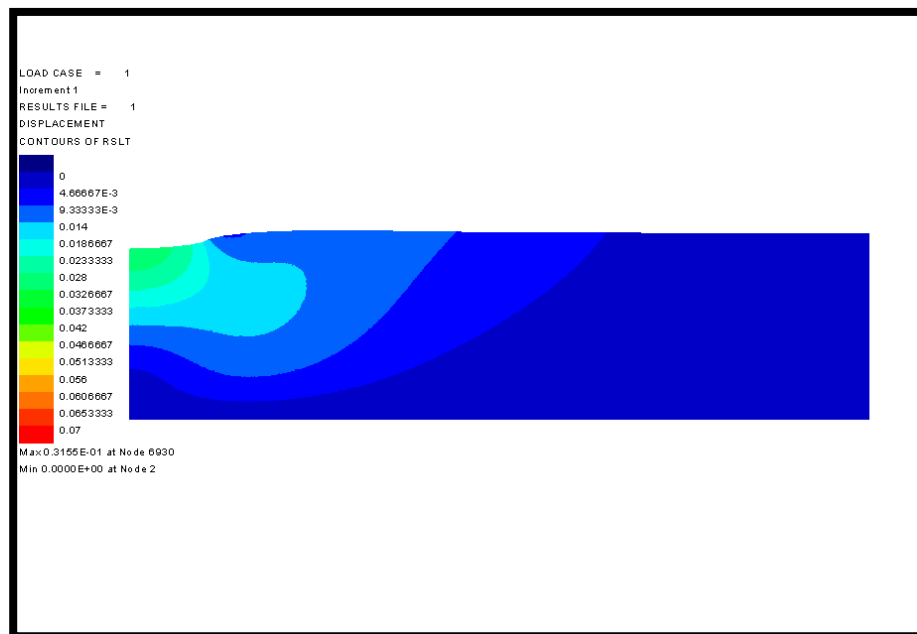
4.2. Undrained analysis

The footing analysis was repeated as an undrained analysis. In this case plane strain elements with additional pore pressure freedoms were used and two-phase material properties were also supplied.

Young's modulus	E'	3000	kN/m^2
Poisson's ratio	ν'	0.25	
Bulk modulus of solid particles	K_s	$1.0\text{E}+15$	kN/m^2
Bulk modulus of water	K_w	$2.1\text{E}+06$	kN/m^2
Porosity	n	0.63	
Unit weight of fluid phase	γ_w	10	kN/m^3
Permeability in x direction	k_x	Not used	m/s
Permeability in y direction	k_y	Not used	m/s
Permeability in z direction	k_z	Not used	m/s

A maximum vertical displacement of 31mm was predicted below the centre-line of the footing.

[Refer to command file *undrained_footing.cmd*]



Displacement contours for undrained footing analysis

4.3. Consolidation analysis

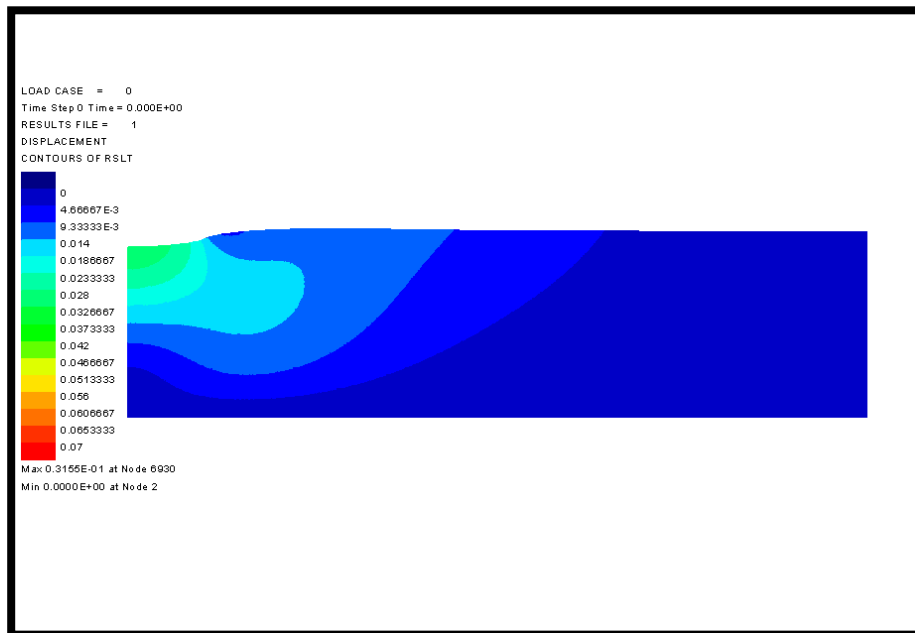
The footing problem was repeated as a full consolidation time-stepping analysis. This required the specification of permeabilities to describe the rate at which water flows through the soil.

Young's modulus	E'	3000	kN/m^2
Poisson's ratio	ν'	0.25	
Bulk modulus of solid particles	K_s	$1.0\text{E}+15$	kN/m^2
Bulk modulus of water	K_w	$2.1\text{E}+06$	kN/m^2
Porosity	n	0.63	
Unit weight of fluid phase	γ_w	10	kN/m^3
Permeability in x direction	k_x	$1\text{E}-08$	m/s
Permeability in y direction	k_y	$1\text{E}-08$	m/s
Permeability in z direction	k_z	$1\text{E}-08$	m/s

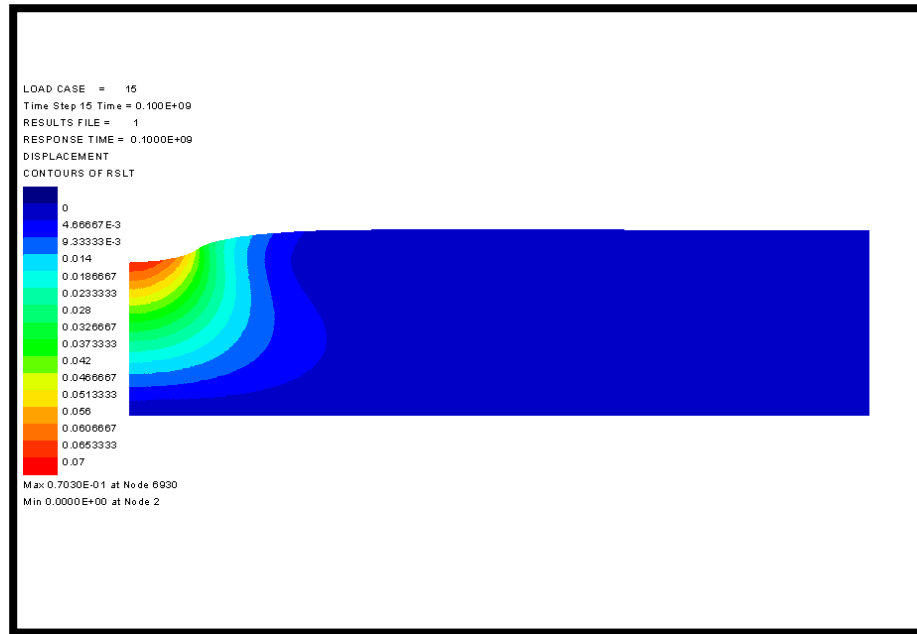
Time steps were chosen which increased exponentially in size up to a maximum of 3 years when almost complete dissipation of excess pore pressures had taken place.

At time $T=0$, just immediately after the load had been applied, the displaced profile agreed with that calculated in the undrained analysis. At $T=3$ years, the displaced profile agreed very closely with that calculated by the drained analysis.

[Refer to command file *consolidation_footing.cmd*]



Displacement contours for consolidation footing analysis at time $T=0$



Displacement contours for consolidation footing analysis at time T=3 years

5. Retaining wall analysis

A simple plane-strain, propped, drained retaining wall analysis was carried out by removing soil elements in front of a reinforced concrete wall to simulate excavation. The model was very similar to one described in the following reference:

POTTS, D. M. & FOURIE, A. B. (1984). "The behaviour of a propped retaining wall: results of a numerical experiment". *Géotechnique* **34**, No. 3, 383-404.

The 1m thick wall was modelled with 8-node quadrilaterals and was assumed to be completely elastic. The wall was 'wished-in-place' in that the installation of the wall itself was not modelled. A rigid prop was assumed to act at the top of the wall by applying a horizontal restraint and excavation took place layer by layer, over a width of 20m, to a depth of 13.26m.

The soil was assumed to be linearly elastic-perfectly plastic with a Mohr-Coulomb yield surface. Strength parameters $c' = 0$ and $\phi' = 25^\circ$ were adopted. Associated flow conditions were assumed in which the angle of dilation ψ is equal to ϕ' . Poisson's ratio was set to 0.2 and Young's modulus

$$E = 6000.Z \text{ (kN/m}^2\text{)}$$

where Z is equal to depth below the top of the wall in metres was used. This was achieved by the use of a variation dataset.

The base of the model was assumed completely fixed while the sides of the model were restrained horizontally.

In situ stresses were specified in the first loadcase by the use of an "initial stress" dataset and also the application of a gravitational bodyforce together with two variation datasets. These two load types enabled the application of an initial stress without causing any significant straining in the soil. Initial stresses with $\sigma_v' = \gamma.Z$ and $\sigma_h' = K_0.\sigma_v'$ were specified where $\gamma = 20\text{kN/m}^3$ and $K_0 = 2.0$, representing an over-consolidated soil.

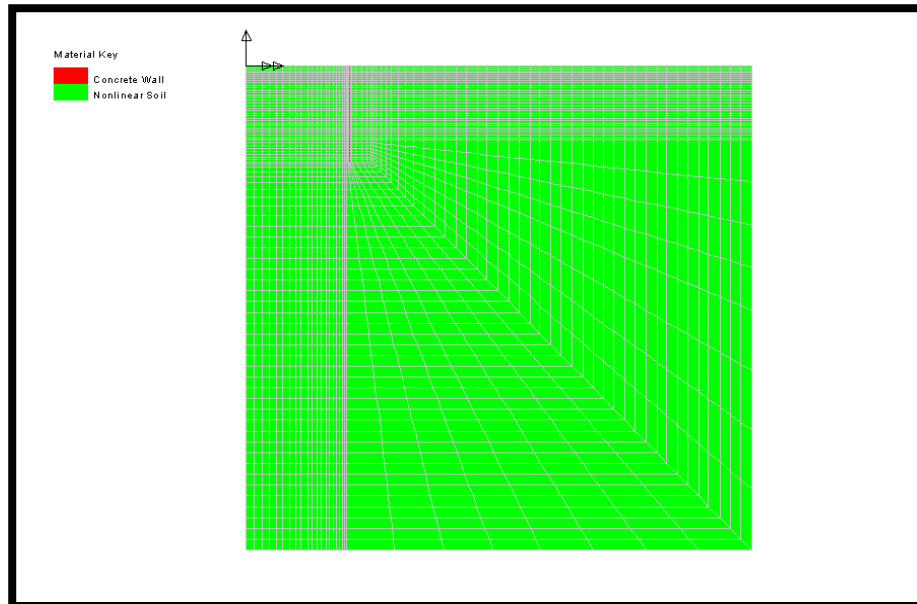
Once the analysis had taken place, horizontal wall displacements and wall bending moments were compared with those presented in the paper. Bending moments within plane-strain elements are not calculated by LUSAS directly and therefore stresses at individual Gauss points were examined and the following formula applied for a 2x2 integration point rule:

$$M_{yy} = 0.14434 \times b^2 \left[-\sigma_{yI} + \sigma_{yII} \right]$$

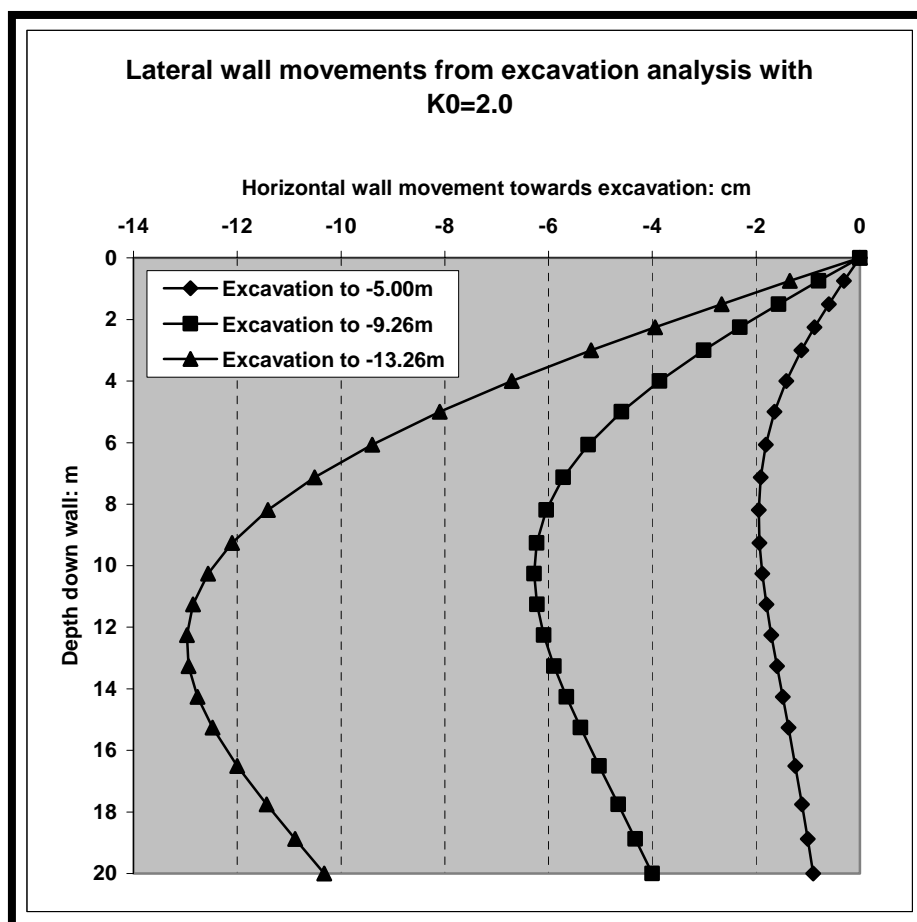
Where b is the width of the wall and σ_{yI} and σ_{yII} are the vertical stresses in the wall at Gauss points each side of the centre-line. Please contact support@lusas.com for a full derivation of this formula.

The results agree well with those presented in the paper.

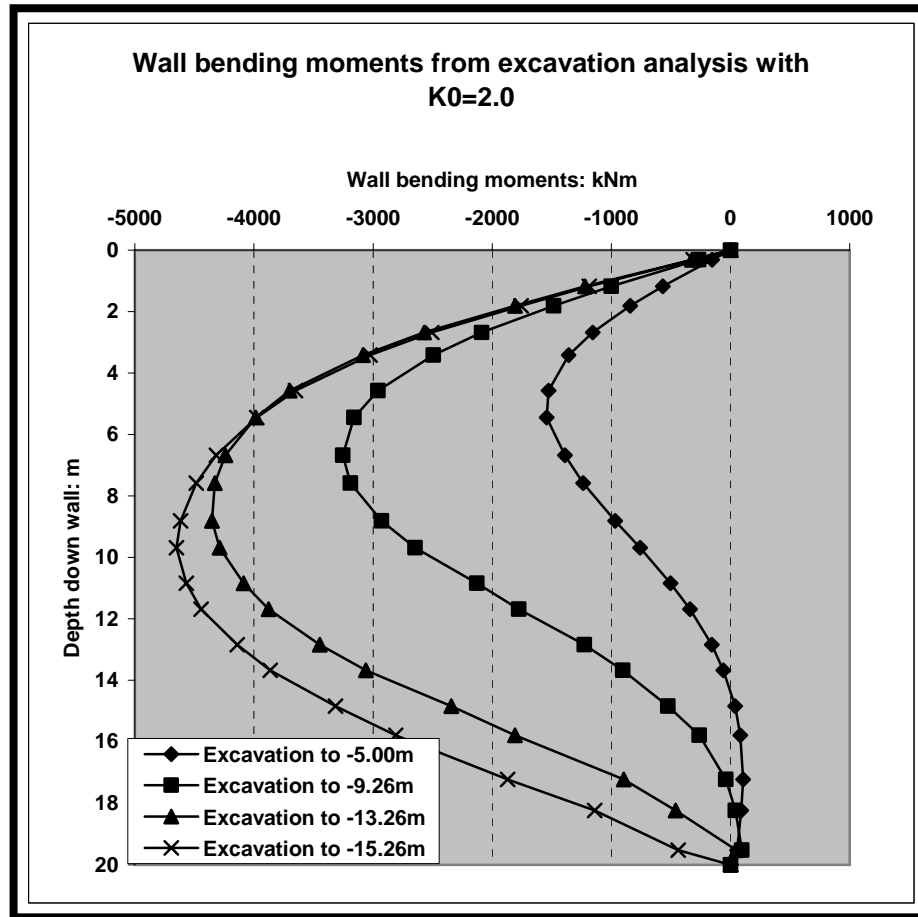
[Refer to command file *drained_wall.cmd*]



Initial retaining wall mesh



Horizontal wall displacements



Wall bending moments