Application of Finite Element Methods to Masonry Bridges

Steve Rhodes, Philip Icke, Paul Lyons

LUSAS, Kingston-Upon-Thames, United Kingdom

Contact: steve.rhodes@lusas.com

Abstract

Many of Europe's oldest bridges – now subjected to traffic unimaginable to the original constructors – are of masonry. Good management of these masonry bridges is demanding. A sustainable approach sees engineers drawing deeply on the available intellectual resources to avoid unnecessary work on the one hand, and unplanned closures on the other. Identifying which structures need strengthening – and planning suitable interventions – requires not merely an assessment of load-carrying capacity but moreover a thorough understanding of the structural behaviour of each bridge.

This paper explores the application of finite element methods to masonry bridges. Options for idealisation are outlined and recommended; considering issues of soil-structure interaction, material parameters and nonlinearity. However, the emphasis is upon promoting an understanding of the structure itself.

Keywords: Bridges; Brickwork & masonry; Stress analysis

1 Introduction

Across the UK and many other parts of Europe, stone and brickwork bridges are vital to the road, rail and communications networks and the majority of these bridges are now more than 100 years old. Vehicle weights and the traffic volumes long ago surpassed those which might have been conceived at the time of their construction and over the years their strength has reduced due a wide range of causes such as action of water, ice, salt, wind, settlement, subsidence, scour, vegetation and human activity [1].

Decisions concerning the service life of a bridge, specification of appropriate monitoring regimes, and any necessary interventions need to be rational and well-informed. For this purpose, robust inspection regimes and records are needed, along with the use of suitable monitoring techniques. Underpinning these decisions must also be a thorough understanding of the behaviour of the structure.

MEXE, equilibrium and mechanism methods are commonly used in masonry bridge assessments. A useful comparison of these methods alongside finite element (FE) and discrete element modelling options in [1] refers to 2D plane strain modelling and 3D FE analysis using curved shell elements, and rightly states that "sophisticated computational methods of analysis are only as good as their input data and the expertise of the assessor". However there is a parallel danger that less sophisticated methods can be used as 'black box' calculators. The 'load-carrying capacity' produced by widely available programs or spreadsheets may sometimes be accepted with unwarranted confidence, considering the uncertainties in the input data and the inherent assumptions.

While placing great importance upon a single 'capacity' calculation may be ill-advised, the construction and validation of analysis models can be very instructive. Deformed shapes and predicted crack locations from analyses should be compared with site observations. The effect of modifying parameters for which values are uncertain should be investigated with a sensitivity analysis. Analyses based on different assumptions should be considered together. The process of comprehending, explaining differences and improving an analytical model promotes a better understanding of the behaviour of the structure in question.

FE software may have extensive facilities. Identifying those which should be applied in order to create an efficient model can be daunting. This paper describes two practical idealisation approaches, with the intention that such models might form a starting point, rather than a final stop, for assessment engineers.

2 Structural idealisation

2.1 Finite element basics

In FE analysis a relatively complex system is divided into elements connected at nodes. For each element, nodal displacements, reactions and internal stresses can be calculated by means of a shape function, based on a stiffness matrix. There are a range of element types, including 'brick' elements, 2D continuum (plane strain, plane stress), 3D shell elements, beam elements and joint elements that can be used in an analysis.

We should first eliminate any particular sense of connection between 'brick' elements and masonry. Brick elements are better referred to as 3D continuum or volume elements, and may be used to model any 3D structure, regardless of the material. Moreover, in the context of masonry bridges it may be far more efficient to use other element types. The selection of element type, which is fundamentally a choice of analysis approach, must be based on a consideration of the structural system, making it the first step in acquiring a better understanding of the bridge in question.

2.2 Structural system and soil-structure interaction

Masonry bridges are typically of unreinforced masonry, a composite material made up of units (bricks, blocks or stones) and mortar. Like mass concrete, it has low tensile strength and historically the demand to carry loads such as those on bridges has been met by arranging the members such that the masonry generally remains in compression.

The masonry components of an arch bridge typically comprise parapet walls resting on spandrel walls resting in turn on an arch barrel sprung from abutments, with wing walls parallel, perpendicular, or at some skew to the abutment walls. Most masonry bridges are backfilled with local material, to provide a near-level running surface, and most include some mass concrete haunching, stiffening the barrel. In some cases the fill is placed between internal spandrels which provide considerable further stiffness and strength. Some longer spans may have voids over, formed by internal spandrels with covering stone slabs or transverse-spanning arches, to reduce foundation pressures.

The behaviour of such structures, describing the participation of the fill as both a load and a part of the support system is set out in [2] section 2. The fill plays a stabilising role, with soil pressures opposing movement in the arch as a vehicle approaches, passes over and moves away from the structure as per [3] section 2.6.3 and [4].

This understanding of the system implies a need to consider the arch and surrounding fill or soil together – a soil-structure interaction (SSI) analysis. However, SSI is not referred to in the widely used MEXE method and is only crudely represented within other common arch analysis methods [4]. FE software provides an alternative in which facilities appropriate to SSI are available.

2.3 Practical FE options

FE models which incorporate SSI can be constructed using 3D continuum elements. However, experience shows that such an approach can lead to an exponential growth in size of model, complexity and solution time since:

1. There is no fixed extent for the model, with soil extending vertically downwards and in both horizontal directions. This can lead to large numbers of elements and nodes in 3D continuum models.
2. Both soil and masonry are inherently nonlinear materials. Solution requires iteration, which is inherently more time-consuming than a simple linear static solution.

For bridge engineers the use of 3D continuum elements is rarely efficient for global analysis – hence the widespread use of beam models, grillage models and shell element models in practice. Turning to masonry bridges specifically, it is proposed in this paper that 2D continuum models and 3D shell element models are the most practical FE options.

3 Continuum modelling in 2D

Figure 1 shows results from a 2D plane strain model, based on the loading to collapse of the Prestwood Bridge, as described in [5]. The soil has been assigned a Mohr Coulomb material and the masonry has been assigned a material which can model cracking and crushing, with the parameters shown in Table 1 – as in [2].

Table 1: Material parameters (Prestwood)

Soil		Masonry	
E'	50E3kN/m²	E	4.14E6kN/m²
ν'	0,25	ν	0,15
ρ	2,04t/m³	ρ	2,04t/m³
c'	7kN/m²	f_c	4,5E3kN/m²
ϕ'	37°	f_t	130kN/m²
Ψ	5°	G_f	0,03kJ/m²

The formation of 4 hinges, in the order numbered (in Figure 1), shows good correspondence to that observed in the test, and the predicted failure load (*P*), at 85% of the test load at collapse (*F*), is quite reasonable. Moreover, the model offers the engineer the possibility of varying properties in order to determine the effects of changes in assumptions on the results. The following sections describe the key components of this model.

3.1 Element selection

Rather than using 3D continuum elements, this model is constructed using 2D continuum elements in consideration of the practical issues of model size described above. Such elements may be either 'plane strain' or 'plane stress' elements. Typically plane strain elements would be most suitable for 2D modelling of the masonry and soil parts of a masonry arch bridge. This effectively assumes that the confinement provided by the spandrel walls and friction provides full resistance to lateral strain in the soil mass. A plane stress analysis would effectively assume no resistance to such lateral strains. Clearly these elements represent the two possible extremes: the true 3D state would be somewhere in-between. With this limitation accepted, this 2D approach considerably reduces the solution time and time spent manipulating the model and extracting results. It may be a limitation worth revisiting once some initial results have been examined.

As in all FE analyses, it is important to ensure that the number of elements used is sufficiently large that any inaccuracy arising from the division strategy may be deemed negligible by comparison to other assumptions inherent in the analysis. The 'discretisation error' can be assessed by comparing key results from several models which are identical except for the number of elements.

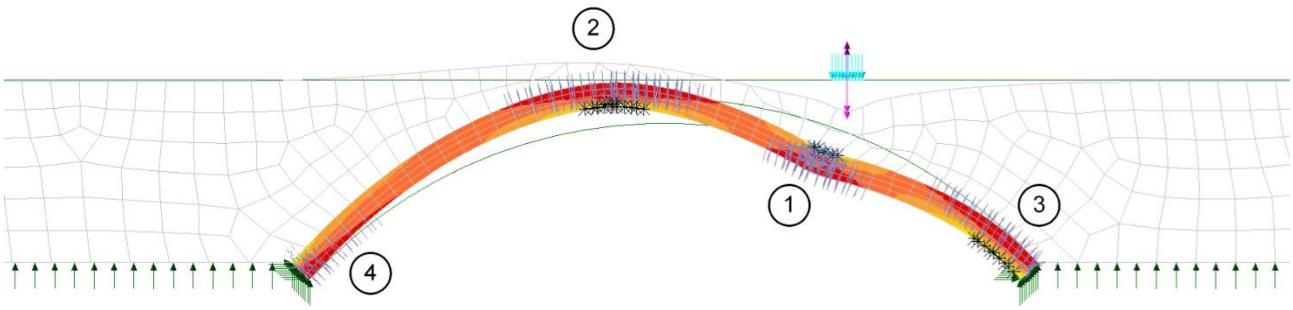

Figure 1. 2D model of Prestwood Bridge at numerical failure; P=0.85×F. Cracking planes shown in grey; crushing indicated with black symbols

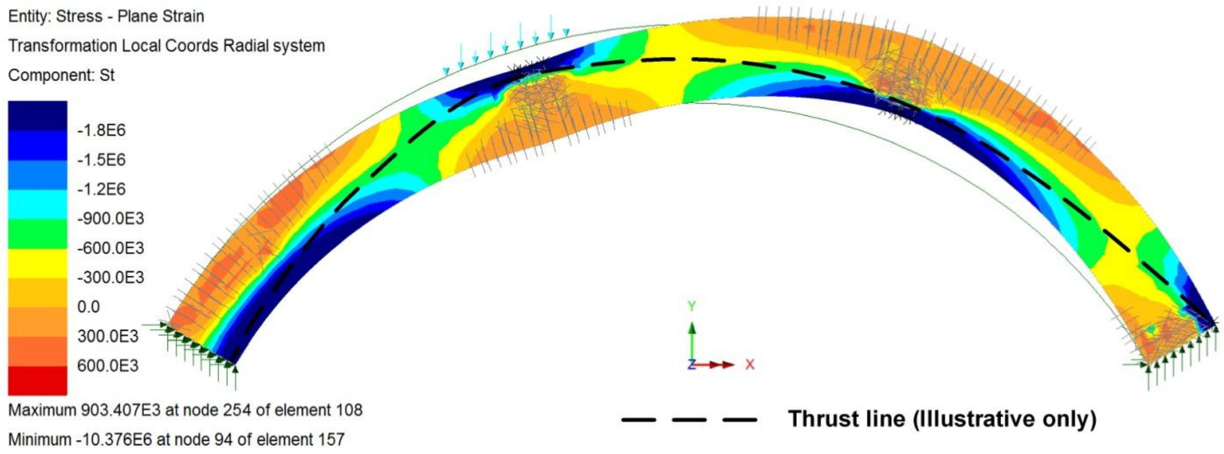

Figure 2. Formation of 4-pinned arch using plane strain model with concrete material

3.2 Soil material nonlinearity

There are many nonlinear material models designed to represent soil behaviour. The Mohr-Coulomb model is widely used as per [3] section 4.4.3.5 and the simple test case in [2] illustrates a correlation correspond with the familiar Rankine-Bell earth pressures.

Determination of input parameters appropriate to an existing bridge is, however, often problematic. Sample values may be obtained from various sources such as [7] section 8.1.4.5 and a helpful summary of the likely influence of values on collapse load is given by [3] section 4.5.8.2. A sensitivity analysis is advisable.

3.3 Interface options

Some studies assume full contact between soil and structure [8], but sliding may take place when approaching collapse, as per [3] sections 4.4.3.5 and 4.5.8.2. Where required, the interface may be represented in an FE analysis by way of [2]:

1. Joint elements & materials. [6] section 4.12.
2. Contact slidelines. [6] section 5.4.
3. Elasto-plastic interface materials. [6] section 4.4.2.2.

3.4 Masonry material nonlinearity

Approaches to the modelling of masonry are also discussed in [2]. The 'smeared' approach is recommended along with a cracking and crushing material such as that described by [9] and [6] section 4.7.

Figure 2 shows how this 'smeared' cracking and crushing material replicates the expected behaviour, as load is applied at approximately quarter-span on an arch barrel.

Determination of suitable values for material parameters is, again, often a challenge for an existing structure. Testing is usually of limited use since obtaining a statistically representative sample would cause damage to the structure. Again [3] provides guidance and sample values, and further references for the example considered are given in [2].

3.5 Large displacements

The cracking behaviour and deformation leads to a displaced thrust line, passing through the uncracked material, as shown in Figure 2 (also refer to [5]). This warrants treatment using large displacement theory as failure is approached – handled by invoking a suitable geometric nonlinear option in the FE solution ([6] section 3)

3.6 Ring separation

Under cyclic loading, the 'fatigue capacity' of multi-ring masonry arches has been found to be of the order 50% of the static strength ([7] section 8.1.4.4). Often these effects are overlooked in bridge assessments.

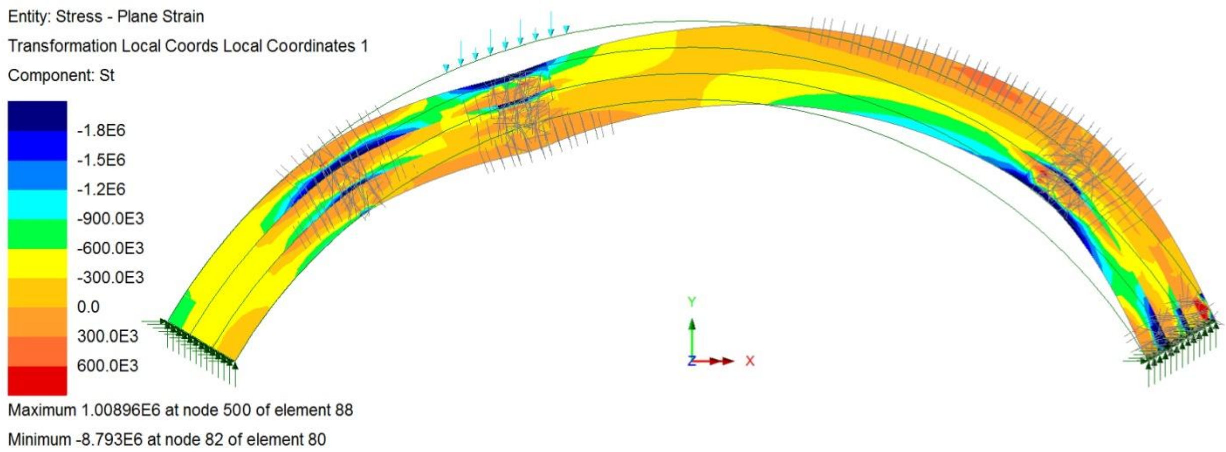

Figure 3. Failure of arch in a model including interfaces to allow ring separation

In the fatigue tests of [10] all the multi-ring arches tested failed by ring separation as opposed to the 4-pin mechanism widely anticipated and illustrated in Figure 2 above. With this in mind, it may be appropriate to model the arch with interface planes between such rings. Figure 3 illustrates the changed behaviour as compared to Figure 2, and corresponding to a significantly reduced ultimate load.

3.7 Support conditions

In general, support conditions have a significant influence on results from bridge analysis, and masonry bridges are no exception. In the examples above, rigid supports have been used, however, in order to obtain realistic stress distributions, sprung supports or explicit modelling of the subgrade using continuum elements is recommended.

When modelling a structure interacting with a soil mass, the extent of the model is not straightforward to define: vertical and horizontal boundaries must be imposed on the soil mass at some distance from the structure. Where such boundaries cannot be reasonably defined to match physical boundaries (e.g. free soil face, bedrock) they need to be determined by comparing key results from several models which are identical except for the assumed width or depth. Where the stiffness of the soil (E') has been assumed constant with depth, the predicted deflection under vertical load at the surface will increase as the depth of soil below the structure is increased so other key results should be used for comparison. If E' increases with depth, this effect is less pronounced.

Settlement, subsidence and scour are identified as important causes of defects in [1]. Support movements cannot be considered by many available tools – a significant limitation when attempting to make sense of the crack patterns or other damage observed in an old bridge. They can, however, be included in FE models.

3.8 Model development and consideration of remedial options

As described, it is likely that there will be much uncertainty in parameters for soil, masonry, supports, and any interfaces. But if the purpose of an FE analysis of a masonry bridge is to promote understanding of the behaviour of the system, then this does not require accurate values for parameters. Instead, it requires a synergistic comparison of model behaviour and results such as crack patterns against observations from site.

Planning of any intervention intended to strengthen a structure must be with great care. When the example of Figure 3 is modified for the insertion of radial dowels, the collapse load may be apparently doubled [2], but the failure mode is found to be more brittle and therefore likely to be more sudden in practice. With this knowledge, a client may prefer to increase monitoring, rather than carry out the intervention.

4 3D shell element modelling

2D models provide a good starting point for validation of a modelling approach using benchmark problems, for the study of SSI effects and sensitivity of the model to assumed parameters, and may in some cases be adequate for the purpose of the analysis. However, it is identified in [3] section 2.1.4 that even "modest span railway bridges often have internal spandrel walls directly below the rails". These are likely to act as stiffeners to the barrel and may have great effect on the behaviour of the structure. The external spandrels and parapets may also stiffen the edges of the arch. No 2D analysis method (including the 2D continuum approach above) can properly take account of the influence which these stiffeners may have upon the structure as a whole. If they cannot be neglected, then a 3D approach is required.

Figure 4 shows results from a 3D model based, again, on the loading to collapse of the Prestwood Bridge [5]. In this instance, the size of model is reduced as compared to a corresponding 3D continuum model, by using shell elements to represent the masonry and joint elements to represent the soil. Further information on transverse behaviour and spandrel wall failure is given in [11].

4.1 Representing the masonry

Shell elements carry in-plane forces and in-plane shears, and transverse loads by flexure, twisting and out-of-plane shears. Crucially shell elements can be formulated in such a way as to allow gradual through-section plastification [6] – or in this case, cracking – enabling them to replicate the softening of masonry due to such damage using a macro-modelling approach. The considerations for modelling the masonry material in the 3D model are then the same as those described for the 2D alternative in section 3.4 above. However, it is not possible for ring separation or the remedial of section 3.8 above to be incorporated in a 3D shell element model. A full 3D continuum model of the bridge would be required

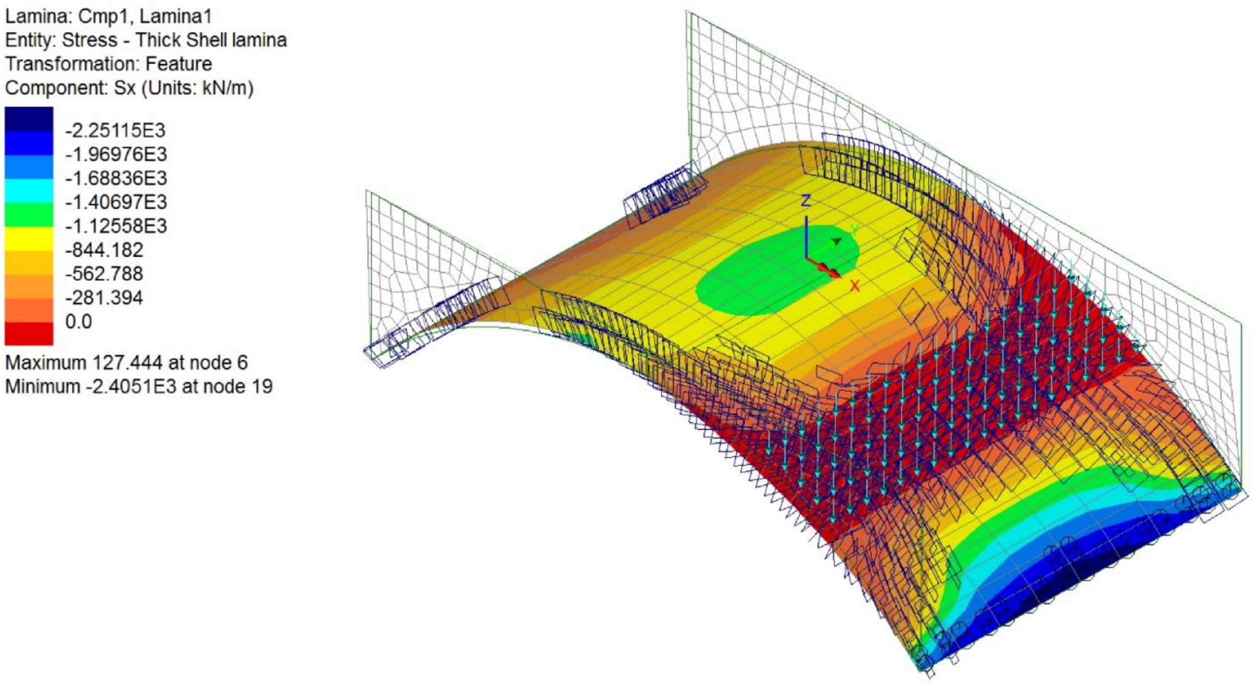

Figure 4. 3D shell model of Prestwood Bridge at numerical failure; P=1.047×F. Cracking planes and crushing locations in underside of arch illustrated.

4.2 Representing the soil

Nonlinear joints have been used in this example, as an alternative to the use of 3D continuum elements, to represent the soil. These joints, acting as springs placed between a notional rigid boundary and the masonry structure (modelled with shell elements) reflect a pressure/ deflection relationship such as that illustrated in Figure 5 below.

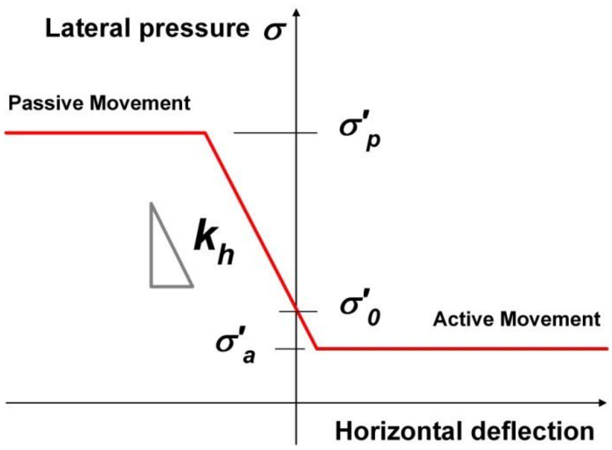

Figure 5. Nonlinear 'soil joint' pressure/ deflection graph (after [6])

Critically Figure 5 incorporates not only the horizontal stiffness of the soil, using a horizontal modulus of subgrade reaction (k_h), but also at-rest earth pressures (σ'_o, based on K_o). Neither quantities are considered when designing structures using limiting earth pressure methods, however they are essential components of SSI analyses. Typically all the quantities represented in Figure 5 – active and passive 'yield points', the spring stiffness, k_h, and the at-rest pressure – vary with depth.

Such an approach is in keeping with EN1997-1 [12] clause 9.5.4 and [5] section 3.4.3(a) and significantly reduces the size of the model, but comes at the cost of particular assumptions:

- the determination of spring stiffness is problematic, since it is not a fundamental property of the soil
- the weight of the soil must be added to the model as a vertical load
- Dispersal of wheel loads must be handled by assumptions such as those of elastic half-space.
- Increase in lateral pressures local to wheel loads is assumed to be negligible.

5 Comparison of results

Both the 2D (plane strain) model and the 3D shell model give a predicted failure load, *P*, which corresponds reasonably to that observed in the physical test: 0.85×*F* and 1.04×*F* respectively. There is then a temptation to assume that this indicates either model to be adequate and even that the shell model is superior. However, it could be that one or both models are exhibiting a false correlation – that is, model behaviour does not reflect the structural behaviour very well, and the numerical agreement is somewhat a matter of chance, perhaps due to erroneous but compensating assumptions. The possible outcomes when comparing two models of the same structure – applicable to all manner of structural analyses – are set out in [13] along with a note that false correlations are surprisingly common.

In this case, the formation of cracks of specific direction and hinges in a specific order was identified as being agreed between the physical test and the 2D plane strain model. On the contrary, the 3D shell model exhibits longitudinal cracking caused by edge stiffening which was not observed in the test. The behaviour of the 3D model is not very well aligned with site observations, indicating that the close agreement of predicted failure load with the collapse load from the test does appear to be a false correlation.

Rather than simply discard the 3D model, more can be learned. It appears that the spandrel walls do not contribute significantly to the behaviour of this bridge. In other structures they may be stronger, stiffer, and deeper, and the contrary would be true. Sensitivity analysis indicates that the soil stiffness does not have a large effect on the behaviour in this case: owing to low depth of fill and low rise in the arch, lateral pressures are not as significant as they may be for other bridges. The inclusion of soil dead weight, with its precompression effects is, however, found to be significant in the calculations. Observations of this sort can assist the engineer in further study of the structure.

6 Conclusions

FE models of masonry bridges can include:

- Explicit modelling of the behaviour of fill including dispersal of load and stabilising effects using nonlinear materials such as Mohr-Coulomb.
- Crushing/ cracking material for masonry, allowing comparison of crack patterns against those observed on site
- Appropriate modelling of soil-structure interface
- Modelling of ring separation to assist in the understanding of possible fatigue failure modes
- Complete flexibility of geometry, materials and support conditions. 3D models may include haunches, internal and external spandrels and incorporate skew as necessary.
- Ability to model defects & repairs

However, any such modelling should be mindful of the limitations imposed by the uncertainty inherent in modelling older structures. FE analysis is a tool, to be used alongside other analysis approaches, understanding their particular strengths and limitations. But most importantly, the results should be brought together with site observations and monitoring, not only to help quantitatively assess, but moreover to promote the necessary understanding of structural behaviour.

7 References

[1] McKibbins LD, Melbourne C, Sawar N, Gaillard C. *Masonry arch bridges: condition appraisal and remedial treatment*. London: CIRIA Report C565; 2006.

[2] Rhodes S, Icke P. Analysis of Existing Masonry Arch Bridges using Finite Elements. In *IABSE Symposium Report, IABSE Madrid Symposium: Engineering for Progress, Nature and People*. 2014:558-565(8)

[3] Union Internationale des Chemins de Fer. UIC 778-3R, *Recommendations for the inspection, assessment and maintenance of masonry arch bridges, 2nd Edition*. Paris: UIC; 2011.

[4] Gilbert M, Smith CC, Hawksbee SJ, Swift GM, Melbourne C. Modelling Soil-Structure Interaction in Masonry Arch Bridges. In *Proceedings of the 7th International Conference on Arch Bridges – Maintenance, Assessment and Repair.* Split:Trogir;2013.

[5] Page J. *Masonry Arch Bridges, State-of-the-Art Review*. London: TRL; 1993.

[6] LUSAS *Theory Manual Volume 1*. Kingston Upon Thames: LUSAS; 2013.

[7] Sustainable Bridges. *Guideline for Load and Resistance Assessment of Existing European Railway Bridges – Advices on the use of advanced methods*. Sweden: COWI; 2007. Available from www.sustainablebridges.net [accessed 26/02/2018]

[8] Andersson A. *Capacity Assessment of Arch bridges with Backfill – Case of the old Årsta railway bridge*. Stockholm: KTH; 2011.

[9] Jefferson AD. Craft—a plastic-damage-contact model for concrete. I. Model theory and thermodynamic considerations. *International Journal of Solids and Structures.* 2003;40:5973–5999

[10] Melbourne C, Tomor AK,Wang J. Cyclic load capacity and endurance limit of multi-ring masonry arches. In *Proceedings of the 4th International Conference on Arch Bridges*, Barcelona:2004;375-385

[11] Wang J, Heath A, Walker P. Transverse Behaviour of Masonry Arch Bridge - Investigation of Spandrel Wall Failure. In *Proceedings of the 7th International Conference on Arch Bridges – Maintenance, Assessment and Repair.* Split:Trogir;2013.

[12] British Standards Institution. BS EN1997-1:2004 *Eurocode 7: Geotechnical design - Part 1: General rules*. London:BSI;2004.

[13] MacLeod IA. Modern structural analysis – Modelling process and guidance. London: Thomas Telford; 2005.