LUSAS CUSTOMER SUPPORT NOTE

Dynamic Analysis Options in LUSAS

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This support note is issued as a guideline only.



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1. Introduction

In recent years, there has been a growing trend towards the construction of lightweight footbridges. Such structures exhibit a reduced mass inertia, which in turns leads to lower natural frequencies, resulting in a greater risk of resonance. Resonance occurs if the frequency of the structure coincides with the frequency of the excitation, e.g. the step frequency of pedestrians and the dynamic forces can cause larger amplitudes of the vibration.

Taking a footbridge as an example, vibrations may occur in vertical and horizontal directions.

Fulfilling the natural frequency requirements that are given in many codes restricts footbridge design: very slender, lightweight structures, such as stress ribbon bridges and suspension bridges may not satisfy these requirements. Moreover not only natural frequencies but also damping properties, structural and non-structural mass and excitation loads (say pedestrian loading) altogether determine the dynamic response. If the vibration behaviour does not satisfy some comfort criteria, changes in the design or damping devices could be considered.

This note concentrates on the methods available in LUSAS to conduct dynamic analyses to assess structures.

There are three methods in LUSAS to obtain information regarding vibration:

- Interactive Modal Dynamics (IMD)
- Interactive Modal Dynamics Plus (IMDPlus)
- Transient Dynamic Analysis (Implicit and Explicit dynamics)

The following comments describe the principal characteristics of these methods and investigate their differences.

2. Eigenvalue Frequency Analysis

2.1 General

An eigenvalue frequency (modal) analysis is the base of vibration analyses as it returns useful information. This can return eigenmodes (mode shapes) and eigen-frequencies (natural frequencies) which are natural properties of the structure and mass participation factors. These results can be used as a basis for other types of vibration (dynamic) analysis.

Because force and damping are not considered in a modal analysis, the absolute values of calculated displacements/stresses of a particular component are not useful for assessment. Moreover, the eigen value results are normalised and again this is a reason that the absolute values of stresses and deformations of a mode shape should not be used in an analysis.

The results of an eigenvalue frequency analysis are important to identify the form of dominant excitations (vertical and/or horizontal). Also, codes of practice suggest frequency limits, that if met, exempt structures from further detailed dynamic analyses.

Note that in all dynamic analyses it is important to have the correct mass on the structure. This consists of the mass of the structural elements and may include added mass from nonstructural features as railings, surfacing etc. The latter may be included by using nonstructural mass elements. Additional mass can significantly change the dynamic behaviour of the lightweight structures and they should not be simulated by extra load as load and mass have different effects in a dynamic analysis.

2.2 Results

After solving the model, mass participation factors are calculated. This can be checked via:

Utilities > Print Results Wizard > Loadcases: Active > Entity: None and Type: Mass participation factors

MODE	MASS PF X	MASS PF Y	MASS PF Z
1	0.135881E-06	0.667773	0.209903E-04
2	0.906751E-02	0.239741E-04	0.810425
3	0.756383E-07	0.140933	0.543227E-08
4	0.213545E-09	0.220458E-02	0.324777E-07
5	0.123108E-06	0.324108E-05	0.431881E-07

The above results show that the first eigenmode is significant for the horizontal Y direction excitation, while the second is significant for the vertical Z direction.

The calculated frequencies of the structure can be obtained via:

Utilities > Print Results Wizard > Loadcases: Active > Entity: None and Type: Eigenvalues

MODE	EIGENVALUE	FREQUENCY	ERROR NORM
1	285.827	2.69074	0.480730E-09
2	666.891	4.11005	0.237275E-09
3	1175.49	5.45670	0.294936E-09
4	2336.65	7.69337	0.851990E-10
5	4199.92	10.3143	0.451069E-06

The user should always check the following to decide on the significance of eigenmodes:

- a) Frequency values
- b) Mass participation factors
- c) Deformed shapes

3. Interactive Modal Dynamics (IMD)

3.1 General

The Interactive Modal Dynamics (IMD) facility within Modeller calculates the modal response of a system to a given input using the eigenvectors from an eigenvalue analysis; it interactively calculates the frequency/time response of a structure. The results forms IMD are the absolute value of structural response and can be sued for assessment (in contrast to eigen value results).

A main feature of IMD is that the excitation is assumed to be stationary. This means that the available excitations will be assigned to the same node(s) in the same analysis.

The results in IMD can be extracted via Modal Expansion (*Utilities > Graph Wizard*) or via an IMD Loadcase (*Analyses > IMD Loadcase*). Modal Expansion through the Graph Wizard will allow obtaining the results at one specific node over a specified time interval or a range of

frequencies, whereas an IMD Loadcase will provide the results in all nodes but at a single specific specified time or excitation frequency.

There are three fundamental assumptions that the user must take into account when using the IMD facility:

- Linearity The system is linear in terms of geometry, material properties and boundary conditions.
- No Cross-Coupling There is no cross-coupling of modes caused by damping. This is
 reasonable as long as the damping of the structure does not exceed 10% of critical
 damping.
- Low Modes Dominant The response is dominated by the lowest few modes.

In IMD if the load is applied at more that one node (but at all nodes with the same frequency) it is possible to use the load locations form a linear static load case as the source of excitation. In such a case choose Excitation: Loading in IMD dialogs.

Utilities > Graph Wizard

lite of Archivers of Bridger	Entity data
Time history	Frequency O Time
Fourier expansion	
Modal expansion	Sample modes Damping
🗇 Load curve	I All Type Specified value 🔻
O Variation	
Specified datasets	Select modes Set damping
Thermal surfaces	Modal excitation
Slidelines (assigned to lines)	Excitation Loading Set

3.2 Damping

In structural dynamics a damping ratio is used to model the actual damping of the structure. Usually, a constant ratio is used for all the eigenmodes of the structure or if more information is provided a different ratio for each of the eigenmodes. The damping values are usually taken from a reference book or codes of practice. However, in reality the damping of a structure cannot exactly be predicted or calculated with the same certainty as it can be done for its stiffness and mass. The reason is that damping is affected by different parameters (for example materials and connections types) and is usually different between eigenmodes. In other words the damping mechanism is not actually known and the values provided from textbooks or codes of practice are based on monitoring similar structures or are a "rough" approximation.

The following table is provided by CEB's bulletin (Comité euro-international du béton) as a guide, but care should be taken as damping is different between service and ultimate loads. The reason for that is that energy consumption is greater in inelastic deformation i.e. when ultimate loads apply.

Type of deck	Critical dan	nping ratio
	Minimum value	Average value
Reinforced concrete	0.8%	1.3%
Prestressed concrete	0.5%	1.0%
Metal	0.2%	0.4%
Mixed	0.3%	0.6%
Timber	1.5%	3.0%

It is suggested to specify different damping values in a parametric analysis to get a range of responses in the model's behaviour, although it is best not to <u>underestimate</u> the damping in order to be conservative.

Damping is specified using percentage viscous (modal) and structural (hysteretic) damping values.

3.3 Results

AMPLITUDE(2) / FREQUENCY(1)

The IMD graph wizard can produce graph of output values which are in interest. For example the following graph shows a frequency vs vertical acceleration for a specific node in the structure. Also, a specific range of frequencies is given in order to monitor the change of the structure's response (acceleration) and spot resonance.

Entity	Displ	acement	2	_	•	Samplir	ng frequ	iency									
Component	DZ				•	Start	0.0										
	Tran	formed	••••	lone		End	7.5										
	Woo	l-Armer				Step	0.1										
Calculated	entity	Acceler	ation		•]												
Туре		Amplitu	de		•	Set	a.]										
Node		815			•												
		< Back		Next	>	Car	ncel		lelp								
1.2		< Back		Next	>	Car	Ac	celeration	l elp /s Freq	Jency a	t 3% d	ampin	1	 	 	 	
1.2 1.1		< Back		Next	>	Car	Ac	celeration	ielp /s Freq	Jency a	t 3% d	ampin	1			 	
1.2 1.1		< Back		Next	>	Car	Ac	celeration	ielp /s Freq	Jency a	t 3% d	ampin	1	 		 	
1.2 1.1 1 0.9		< Back		Next		Car	Ac	celeration	ielp /s Freq	Jency a	t 3% d	ampin		 	 		
1.2 1.1 1 0.9 0.8		< Back		Next		Car	Ac	celeration	ielp vs Freq	Jency a	t 3% d	ampin				 	
1.2 1.1 1 0.9 0.8 0.7		< Back		Next		Car	Ac	celeration	ielp /s Freq		t 3% d	ampin					
1.2 1.1 1 0.9 0.8 0.7 0.6		< Back		Next		Car	Ac	celeration	ielp vs Freq	Jency a	t 3% d	ampin					
1.2 1.1 0.9 0.8 0.7 0.6 0.5		< Back		Next		Car	Ac	celeration	ielp vs Freq	Jency 8	t 3% d	ampin					
1.2 1.1 1 0.9 0.8 0.7 0.6 0.5 0.4		< Back		Next		Car	Ac	celeration	ielp //s Freq	Jency z	t 3% d	ampin					
1.2 1.1 1 0.9 0.8 0.7 0.6 0.5 0.4 0.3		< Back		Next	>	Car		celeration	rs Freq	Jency #	t 3% d	ampin					



4. Interactive Modal Dynamics Plus (IMDPlus)

4.1 General

The Interactive Modal Dynamics Plus (IMD) facility allows the definition of a constant moving load across a pre-specified path and obtains the results for a specified range of travelling speeds. This is therefore an excellent tool to represent moving loads such as vehicles (road and railway) which are moving along the structure.

It is also possible to analyse a moving spring and mass system in IMDPlus to simulate the effect of a vehicle mass and its suspension system during the movements.

uble sprung mass 🔄		Unsprung mass	Sprung mass 1	Sprung mass 2
Damping	Mass Viscosty coefficient			
Nonlinear springs Polynomial Piecewise linear	1			
		Spring 1 stiffness	5pr	ing 2 stiffness
Ks2 Ks2 Ns1 Cs2 Ks1 Cs1 Cs1	Polyncmial constants	Constant	1	Constant
Name				(new)

There is a detailed IMDPlus Manual with some examples at the following location on the LUSAS installation folder:

".. \LUSAS151\PDF Manuals\EnglishUS\IMDPlus User Manual.pdf"

It is strongly recommended to read this User Manual and try the supplied examples in it before using the IMDPLus.

The IMDPLus approach differences with the previous approach in IMD are the following:

- Moving not stationary load
- Constant load or moving mass-spring system not pulsating load
- Possibility of Calculating the dynamic effects for different speeds

When using IMDPlus moving load the four steps in the dialog should be followed in the logical order.

- 1- Define the vehicle
- 2- Move the vehicle along the path
- 3- Compute Modal forces
- 4- Get the final moving load analysis results.

Note: Options are greyed out if they are not available yet.

<u>M</u> esh Annotation Iransformation Variation	H △ S • F	↓ • • • • • • • • • • • • • • • • • • •
Reference Path Objects to Drape Response Spectrum		
IMD <u>P</u> lus	Seismic	
Design Factors	Moving Load	Vehicle Configuration
	Moving Mass >	Moving Load Generation
Set <u>Fourier Angle</u> Set Harmonic Component		Modal Force Calculator

A line across the span of the bridge is used as a path for the moving load. The load is then used to compute modal forces for the IMDPlus analysis and finally the response of the structure is calculated.

Ensure that a search area is assigned onto the model if needed. This is the case for most 3D models.

Select the path line and enter the IMDPlus dialog:

In the following dialog damping and speed parameters are input through the Moving Load Analysis Control dialog.

Moving load input								
Vehicle configure (Composite axle)	ation e)	1:VML1						*
Mode/damping contr	ol				Tot	al particin	ating mas	5
Included modes:		All		Modify	X-E	Direction	99.99	%
	-	5.0		Madit	Y-E	Direction	100.00	%
Default damping		5.0	%	Modity	Z-C	Direction	99.99	%
Solution control								
Advanced solution	options	•		Modify				
Minimum speed	5			Quiet time after passage of load	G 0	.0		
Maximum speed	25			🔽 IMDPlus au	tomatic	time step	(Nyquist)	6
Speed increment	5			Solution time st	tep			

The quiet time is the time to be added to the solution to allow the decay of the structural response to be analysed and visualised, if necessary. If zero is entered, the IMDPlus analysis will finish immediately after the load has reached the end of the path. A sufficient time step must be entered to capture the response of the structure.

For example, the response time history graph (vertical acceleration) of a specific node can be produced. This will help to assess the structure and find out at which position of the moving

load the acceleration takes its peak value(s).

ode/elemer	t selection			
Node	Element	Entity	Acceleration	
Extent	Al	Component	AZ	+
Node	-		Individual items	
			Sum of chosen ite	ms
			V Average of choser	n items
utput contro	bl			
Respon	se time history			
🗸 Peak re	sponse summary		Positive/negative	Envelope
			Absolute	
	ary response spectra	(SRS)		Modify
Second				
Second	ombination/factor his	tory		Options
Second Modal c Genera	ombination/factor his te graphs in Modeller	tory	Speed to graph	1:5.0 •
Second Modal c Genera Genera	ombination/factor his te graphs in Modeller te textfile output	tory	Speed to graph	Options 1:5.0 Options



If Modal Combinations option is selected it is possible to get the overall response of the whole structure at any time as shown below. For details see the IMDPlus User Manual:

".. \LUSAS151\PDF Manuals\EnglishUS\IMDPlus User Manual.pdf"



Vertical displacement plot for a time of 1.0s

5. Transient Dynamic Analysis

5.1 General

Transient dynamic analysis can be used to monitor the response of the structure over time. Moreover, nonlinearities such as geometric, material and boundary conditions may be included unlike IMD(Plus) analyses which are strictly linear.

Transient Dynamic analysis (step by step dynamics) is time consuming due to its nature and results' accuracy may be dependent on the time step used i.e. the interval of time for which results are obtained. The selection of time step is crucial as it can either lead to extensive solution times or ambiguous results.

When analysing a structural dynamics problem, in most cases, an implicit unconditionally stable time integration is most effective. Then, the time step Δt need generally be only $T_{co}/20$ where ω_{co} would be the highest frequency to be included in the solution [Dynamics of structures" – R.W.Clough, J.Penzien]. Thus, from the eigenvalue analysis an eigenvalue with a large mass participation factor (e.g. 80%) is selected and its period or frequency is used to calculate a time step estimate with a value of $T_{co}/20$ to $T_{co}/30$.

For large models with many time steps it is recommended to use LUSAS restart facility in order to keep file sizes down. Finally, in case that one of the restart files fails to converge for some reason, the user does not have to start the analysis from the beginning, but it can be continued from the failing stage. Contact LUSAS Technical Support <u>support@lusas.com</u> for more information on restart files if needed.

5.2 Damping in Transient Dynamics

Damping needs to be included into the material properties in any transient dynamic analysis. In order to calculate the Rayleigh damping parameters a spreadsheet is provided on the LUSAS User Area. For Rayleigh damping the lower and higher values in the range of frequencies in interest should be used. See

http://www.lusas.com/protected/instruct/damping.html

Plastic	Creep	🕅 Damage	Shrinkage	Viscous	Two phase	
Elastic						
	Dyn	namic properties	2		Value	
	V The	rmal expansion	Young's modulus	2.	0.0E6	
			Poisson's ratio		0.3	
			Coefficient of thermal ex	mansion 1	20F-6	
			Mass Rayleigh damping	constant 0.6	13043	
			Stiffness Rayleigh damp	bing constant 1.4	0415E-3	
	Name Iso1			•	1)	
	Name Iso1			•	1)	
	Name Iso1			 ▼ × 	1)	
	Name Iso1		Close	▼ ★ (▼)	1) Help	
	Name Iso1		Close	✓ ★ (▼ ▼	D Help	
	Name Iso1		Close	Apply	1) Help	
	Name Iso1	Ravi	Close Ca		1) Help	
20;	Name Iso1	Rayl	Close Cc	ancel Apply	D Help	
ζe: put the start a	Name Iso1	Rayl	Close Ca leigh Damping Coef		D Help	
ze: put the start a mut the damni	Name Iso1	Rayl	Close Ca	ance Apply	1) Help	
ge: iput the start a iput the dampie tract the Fau	Name Iso1	Rayl es of interest at these frequencies efficients and neacht	Close Cc leigh Damping Coef	ance Apply	D Help	
ze: put the start a put the dampin stract the Ray	Name Iso1 nd end frequencia ng ratios required leigh damping coo	Rayl es of interest at these frequencies efficients and specify	Close Ce eigh Damping Coef	ance Apply ficents	р Нер	
ge: iput the start a iput the dampi xtract the Ray	Name Iso1 nd end frequencie ag ratios required leigh damping coo	Rayl es of interest at these frequencies efficients and specify Frequency (F12)*	Close Ca eigh Damping Coef in the material propert	ance Apply	1) Help	Figurent
ge: put the start a put the dampi stract the Ray	Name Iso1 nd end frequencie ag ratios required leigh damping coo	Rayl es of interest at these frequencies efficients and specify Frequency (Hz)* 2.6907	Close Coefficient	ance Apply ficents ties of the material Damping (%)	1) Hep	Eigenvalu 20133
ge: put the start a put the dampi ktract the Ray	Name Iso1 nd end frequencie g ratios required leigh damping coo	Rayl es of interest at these frequencies efficients and specify Frequency (Hz)* 2.6907 4.1101	Close Cc eigh Damping Coef in the material propert Damping Ratio* 0.03	ancel Apply Ficents ties of the material Damping (%)	D Help Circular Frequency (/s)	Eigenvalu 265 89 005 89
ge: put the start a put the dampin trract the Ray	Name Iso1 nd end frequencia ng ratios required leigh damping coo lst 2nd	Rayl es of interest at these frequencies efficients and specify Frequency (Hz)* 2.6907 4.1101	Close Ce eigh Damping Coef in the material propert Damping Ratio* 0.03 0.03	ancel Apply Ficents ties of the material Damping (%) 3 3	D Hep Circular Frequency (/s) 10/01 20:52	Eigenvalu 287-86 066-89
ge: put the start a put the dampi ktract the Ray	Name Iso1 nd end frequencie ag ratios required leigh damping coo 1st 2nd	Rayl es of interest at these frequencies efficients and specify Frequency (Hz)* 2.6907 4.1101	Close Ca eigh Damping Coef in the material propert Damping Ratio* 0.03 0.03	ancel Apply ancel Apply ficents ties of the material Damping (%) 3 3 3	1) Help Circular Frequency (/s) 16-91 27:32	Eigenvalu 245-88 066-89
ge: uput the start a uput the dampi tract the Ray	Name Iso1 nd end frequencie ag ratios required leigh damping coo 1st 2nd	Rayl es of interest at these frequencies efficients and specify Frequency (Hz)* 2.6907 4.1101 Rayleigh Damping P	Close Cc leigh Damping Coef in the material propert Damping Ratio* 0.03 0.03	ancel Apply Ficents ties of the material Damping (%)	D Help Circular Frequency (/s) 16.01 25.02	Eigenvalu 285.85 046.59
ge: put the start a put the dampin trtact the Ray	Name Iso1 nd end frequencia ag ratios required leigh damping coo 1st 2nd	Rayl es of interest at these frequencies efficients and specify Frequency (Hz)* 2.6907 4.1101 Rayleigh Damping P.	Close Ce leigh Damping Coef in the material propert Damping Ratio* 0.03 0.03	ancel Apply Ficents ties of the material Damping (%)	D Help Circular Frequency (/s) 10.21 21.32	Eigenvalu 2005-05 0000-05

5.3 Varying Load – Loadcurve

A loadcurve can be used to model a stationary pulsating load. The loadcurve can either be described piecewise or a standard curve can be used.

🔘 User-defined		Standard curve		🕐 Variation		
	Time	Factor	Туре	Sine 💌	Termination time	0.0
1			Amplitude	1.0	Sampling increment	0.0
			Frequency	4.11005		
			Phase angle	0.0 •		
		Activation time	0.0	Scaling factor	1.0	
Nam	ne Load cu	rve 1				

For example a stationary crowd loading can be defined as a standard sinusoidal function. The loadcurve can be visualised using the Graph Wizard to check that it is properly defined.

Utilities > Graph Wizard

Select '*Loadcurve*' and click '*Next*'. On the next dialog select the loadcurve number and click '*Next*'.



5.4 Nonlinear and Transient settings

It is important to allow enough time in a transient dynamic analysis to obtain a steady-state response of the structure. The required time is not known in advanced, but the highest period of the structures can be used to obtain an initial guess.

As discussed earlier, this type of analysis require extended solution times and produces large file sizes, thus it's generally suggested that the restart facility is used or amend the Nonlinear & Transient controls not to store results at every time step by amending "Plot file" value in Incremental LUSAS file output section.

Incremental LUSAS file output	
Same as previous loadcase	
Same as previous loadcase	
Output file	1
output life	-
Plot file	5
	_
Restart file	0
Max number of saved restarts	0
Log file	1
History file	1

5.5 Results

The graph of output quantities in interest can be drawn using Utilities > Graph Wizard. For example a total response time vs vertical acceleration for a specific node in the structure. Steps which are required in *Utilities* > *Graph Wizard* are shown below.

h Wizard	Time History Graph - X Attribute
ype of graph dataset to generate Ime history Fourier expansion Modal expansion Load curve Variation Specified datasets Thermal surfaces Sildelines (assigned to lines)	Results Entity data Nodal Gauss point Element Previously defined Select Select Select
< Back Next > Cancel Help	<back next=""> Cancel Help</back>
listory Graph - X Attribute	Time History Graph - Y Attribute
lamed data Data Response time →	Results Entity data Nodal Nodal Named Element Previously defined History files Select
< Back Next > Cancel Help	< Back
Vocal data Selected nodes Entity Acceleration Component AZ Transformed None Wood-Armer Extent Specified single node Soecify node 815	Vispiay X scale Oreate new graph Title Acceleration vs time with 3% da x Time (sec) Y Acceleration (m/sec2) Acceleration (m/sec2) Add to existing graph On max 1.0 Time Do not directly as any
< Back Next > Cancel Help	Image: Show grid Image: Show symbols Image: Show symbols Image: Show grid Image: Show symbols Image: Show symbols Image: Show grid Image: Show symbols Image: Show symbols Image: Show grid Image: Show symbols Image: Show symbols Image: Show grid Image: Show symbols Image: Show symbols Image: Show grid Image: Show symbols Image: Show symbols Image: Show grid Image: Show symbols Image: Show symbols

The following graph shows that after around 5 seconds the steady-state harmonic response is achieved and the maximum acceleration is approximately 1.07m/sec², which is almost identical to the result the IMD loadcase returned for this linear example.



5.6 Pedestrian load – script

The previous example was with the assumption of a stationary load, however if a moving load (for example pedestrian load) is intended then a customised script can be used to move and vary the load in a transient dynamic analysis.

See: http://www.lusas.com/protected/download/scripts.html

A moving pulsating load according to EN1991-2:2003 has been included in a script on the above LUSAS User Area page. In this case the load is defined with a sinusoidal function and while it's moving it takes different values at each time step. The following screenshot shows the input dialog for this script.

C 💌	
Walking	•
76	
4.11005	
0.35	
0.18858	
1:Srch1	•
0)	
0.01	
steps = 2900	
	76 4.11005 0.35 0.18858 1:Srch1 e 0) 0.01 steps = 2900

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The load path could be extended outside the actual span of the bridge to allow the bridge to vibrate after the load has been removed.

5.7 Results

Following the same approach as before, by using Graph Wizard a total response time vs. vertical acceleration graph for a specific node in the structure can be created.



6. Appendix A

In order to calculate the applying pedestrian load and make the example more realistic the NA to BS EN1991-2:2003 is followed; the assumptions made, though, are only for demonstrative reasons.

6.1 Vertical pulsating stationary load

A vertical pulsating distributed load is given from the formula

$$w = 1.8 \cdot \left(\frac{F_o}{A}\right) \cdot k(f_v) \cdot \sqrt{\gamma \cdot \frac{N}{\lambda}} \cdot \sin(2 \cdot \pi \cdot f_v \cdot t) = 9.576 \cdot \sin(2 \cdot \pi \cdot f_v \cdot t)$$

The reference load F_o is given in Table NA.8 and for walking pedestrians equals to 280N with a crossing speed v_t of 1.7m/sec.

From Figure NA.8 the pedestrian combined factor $k(f_{\nu})$ for 4.1105Hz is 0.35

For a Class C bridge table NA.7 gives a crowd density of 0.8 persons/m² and for an area of $94.6m^2$ the number of pedestrians in the group N is 76.

A damping ratio ξ of 3% will give a logarithmic decrement δ of 0.18858 according to the following formula:

$$\delta = \frac{2 \cdot \pi \cdot \xi}{\sqrt{1 - \xi^2}}$$

Then, from Table NA.9 reduction factor γ is 0.22

Finally, the effective number of pedestrians λ is conservatively taken as 0.634.

By inputting the above values into the formula we get:

$$w = 1.8 \cdot \left(\frac{280}{94.6}\right) \cdot 0.35 \cdot \sqrt{0.22 \cdot \frac{76}{0.634}} \cdot \sin(2 \cdot \pi \cdot f_v \cdot t) = 9.576 \cdot \sin(2 \cdot \pi \cdot f_v \cdot t)$$

Thus, the amplitude is 9.576N/m².

6.2 Vertical pulsating moving load

A vertical pulsating force $\mathsf{F}(\mathsf{N}),$ moving across the span of the bridge at a constant speed $\mathsf{v}_t,$ is calculated by:

$$F = F_o \cdot k(f_v) \cdot \sqrt{1 + \gamma \cdot (N - 1)} \cdot \sin \left(2 \cdot \pi \cdot f_v \cdot t\right)$$

Using the values in 7.1 we get:

$$F = 280 \cdot 0.35 \cdot \sqrt{1 + 0.22 \cdot (76 - 1)} \cdot \sin(2 \cdot \pi \cdot f_v \cdot t) = 410 \cdot \sin(2 \cdot \pi \cdot f_v \cdot t)$$

Thus, the amplitude is 410N.

7. References

- BD37/01
- EN1991-2:2003
- "A design guide for footfall induced vibration of structures"- M.R.Willford, P.Young The Concrete Centre
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