



Online training course

## Session 1

# Introduction to Grillage Analysis

*Session will start on the hour*



Session 1

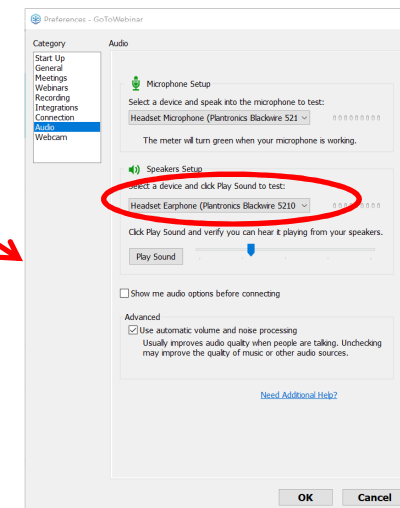
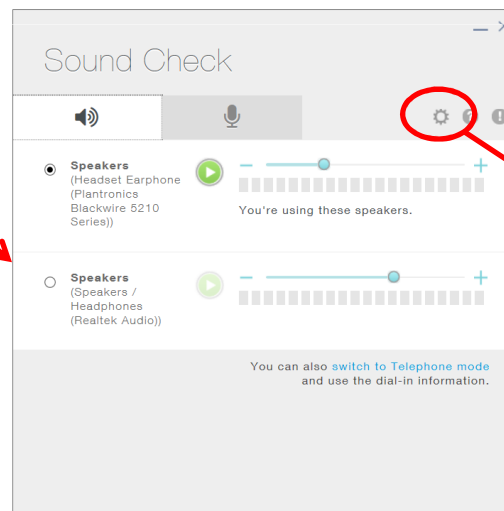
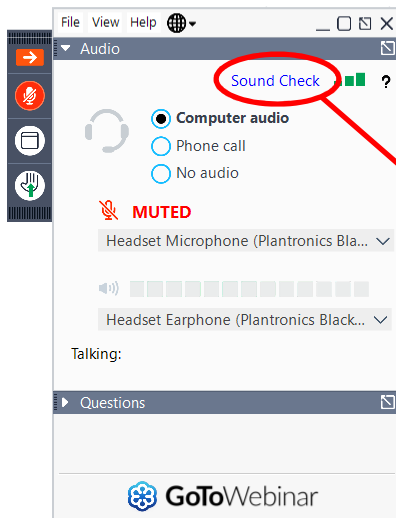
# Introduction to Grillage Analysis

*Presented by:*

*Julian Moses* CEng MIStructE

# Audio Settings

- We are now talking
- If you can't hear us, please click Sound Check, then the Settings cog, then select your speakers



# Audio Settings

- If you still can't hear us, please be aware that a recording of this session will be made available afterwards
- But we're now going to move on with the session for the benefit of the majority of trainees who have working audio

# Schedule

- 12<sup>th</sup> October      **Introduction to Grillage Analysis**
- 19<sup>th</sup> October      **Advanced Grillage Analysis**



# Training Format

- 2 hour session
  - Presentation on basic LUSAS grillage features
  - Demonstration of LUSAS on basic grillage
  - **Type questions into the Questions or Chat box**
  - Session will be recorded
- Homework
  - Worked Example for you to complete (see handouts)
  - You will need the latest version loaded (20.0)
  - Support available via email ([onlinetraining@lusas.com](mailto:onlinetraining@lusas.com))
  - Quiz to test your understanding



# Training Format

- Installation/Licenses
  - You don't need LUSAS installed or a license to watch this session
  - You do need it installed with a license to do the homework!
  - If you don't already have one, please contact your local LUSAS expert and ask them to contact their LUSAS Account Manager



# Grillage modelling

- Introduction to grillages
- Creating a grillage model
- Tips for
  - Flat slab grillages
- Loading, supports etc
- Results
- Skew slab example



# Structural idealisation



THE INTERNATIONAL ASSOCIATION FOR THE ENGINEERING ANALYSIS COMMUNITY

*“There is a strong tendency for new users of FE to focus on the generation of an accurate geometric model ...*

*Powerful and alluring graphics ... and facilities to import geometry from 3D CAD packages reinforce this tendency.*

*The purpose of a finite element analysis is to **model the behaviour of a structure under a system of loads, not its geometry.**”*



How to model with Finite Elements, Chapter 2

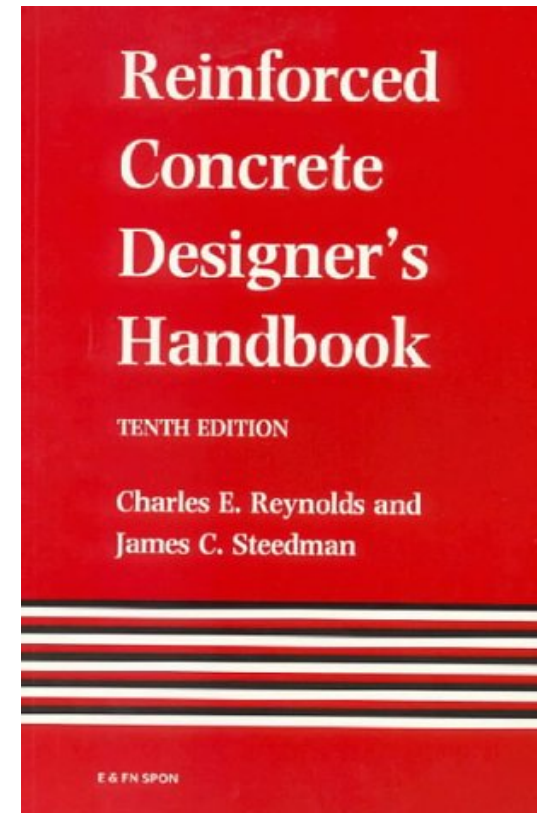
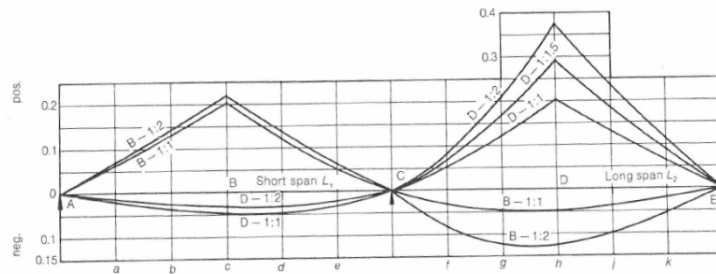
# Structural idealisation

## 1) Approximate/Hand methods

- Once used extensively for preliminary design work
- Now that computers/software are widely available the same method is used for both preliminary and detailed design
- Approximate methods based on statics are still important for checking, likely magnitude of the principal effects before embarking on detailed analysis

# Structural idealisation

## 1) Approximate/Hand methods



# Structural idealisation

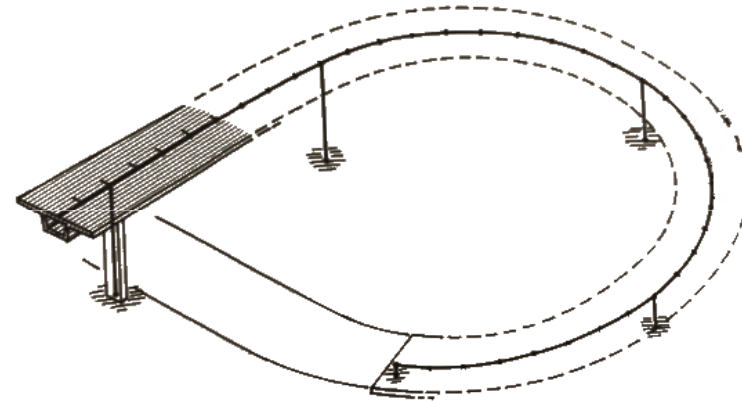
## 2) Line beam model

- Structure is represented as a single member, 2D for a straight structure and 3D if curved
- Overall longitudinal bending moments, shear forces, torsions and reactions etc
- Assumes that all parts of the cross section are involved in resisting the applied loads

# Structural idealisation

## 2) Line beam model

- ✓ Fast
- ✓ Section properties easy
- ✗ Transverse effects
- ✗ Live load distribution
- ✗ Diaphragms/Bracing?



# Structural idealisation

## 2) Line beam model

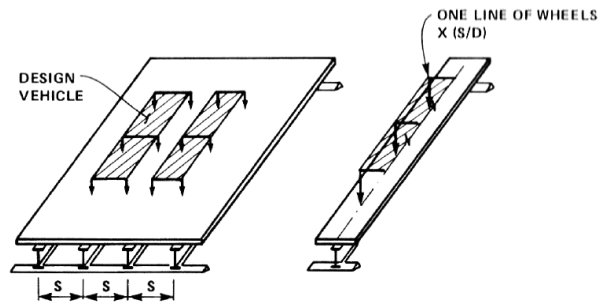
- Simplest types of structure such footbridges
- Spine structures such post tensioned box bridges
- Road bridges that are wide can be analysed this way with the use of live load distribution factors
- **Not numerically rigorous and can be overly conservative**

# Structural idealisation

## 2) Line beam model

*“Refined methods of analysis may reduce the midspan moment by 18 to 23% in the case of interior I-beams, and by 4 to 12% for exterior I-beams when compared to the LRFD simplified method.”*

Section 7.6.2



# Structural idealisation

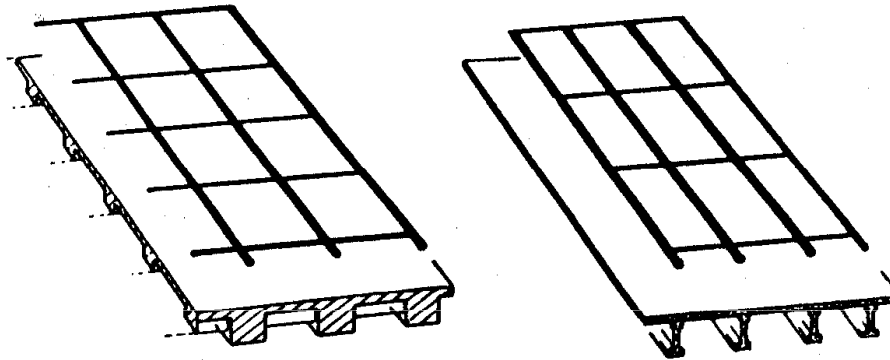
## 3) Grillage (Grid)

- The real structure is represented by a model composed of discrete beam elements spanning longitudinally and transversely
- The method can be used for all types of bridges, although it is more difficult to apply to boxes



# Structural idealisation

## 3) Grillage (Grid)



- ✓ Wizard = fast
- ✓ Section properties = fast with section calculation facilities
- ✗ Bracing?
- Construction cases?
- ✗ In-plane effects? (Integral abutment bridges)

# Structural idealisation

## 3) Grillage (Grid)

- The main advantage of grillage modelling is in the direct availability of composite load effects
- Does not allow properly for shear lag, although an approximate allowance can be made
- High skews introduce problems with grillage layout and accuracy

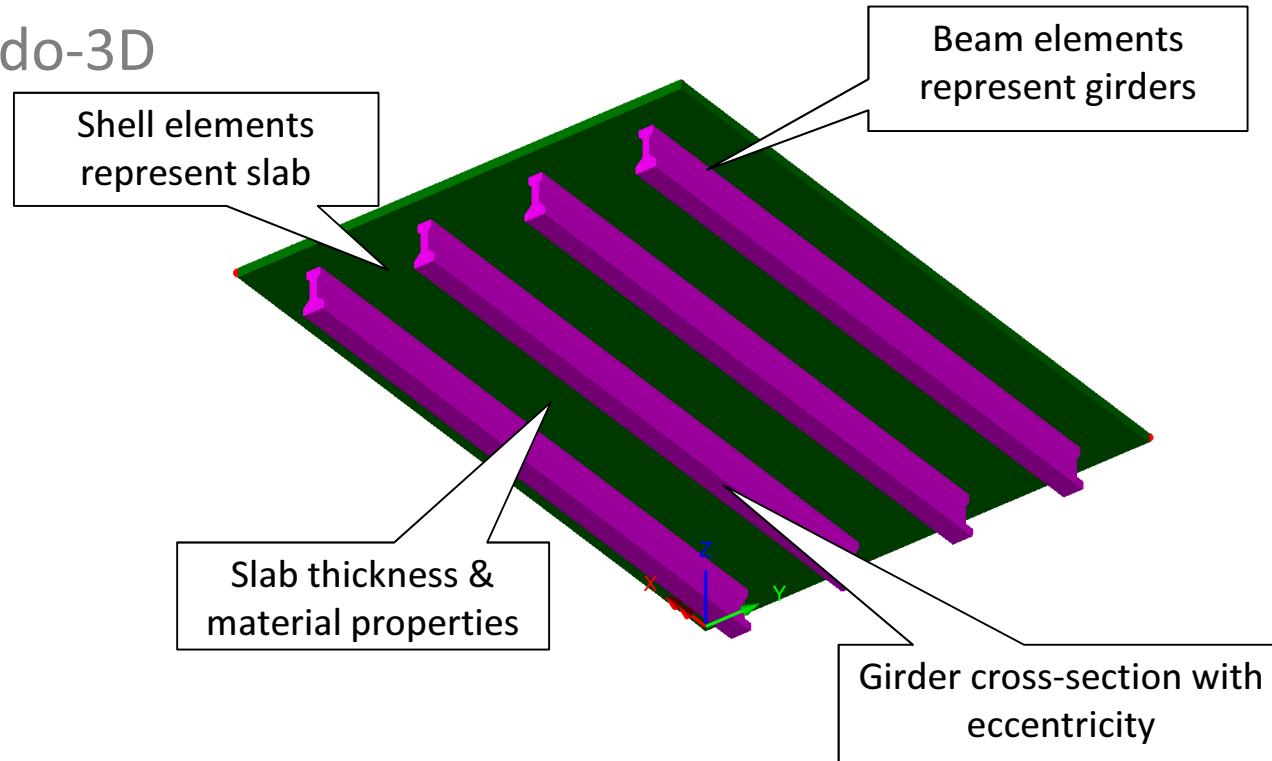
# Structural idealisation

## 4) Pseudo-3D

- The real structure is represented by a model composed of discrete beam elements spanning longitudinally and shell elements transversely
- Elements connected by shared geometry
- The method can be used for bridges that do not have significant bracing along the span
- Difficult to apply to boxes

# Structural idealisation

## 4) Pseudo-3D



# Structural idealisation

## 4) Pseudo-3D

- Since composite section is made up of multiple elements, composite effects only available by slicing or further post processing
- Would not recommend to be used for integral bridge
- ✓ Fast , section properties straightforward
- ✗ Cross-bracing?
- ✓ Local & global effects
- Composite and Non-Composite Moment available
- ✓ Construction cases readily created/ analysed

# Structural idealisation

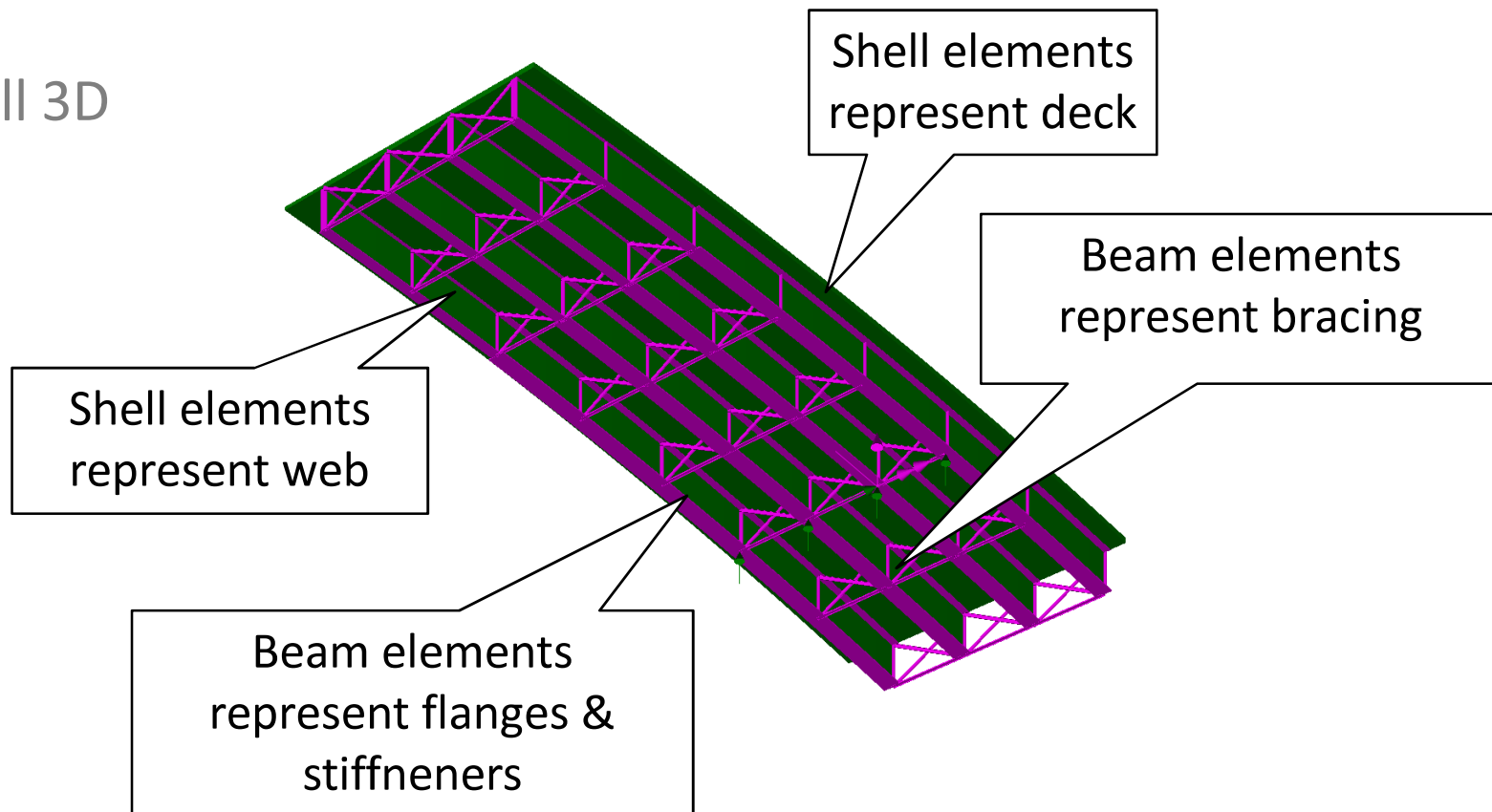
## 5) Full 3D

- The real structure is represented by a model composed of beam and shell elements
- Very powerful, general method of analysis that can be applied to almost any form of structure or structural



# Structural idealisation

5) Full 3D



# Structural idealisation

## 5) Full 3D

- ✓ Shear lag in slab modelled explicitly
- ✓ Cross-bracing can be incorporated
- ✓ Direct visualisation of model
- ✓ Local & global effects
- ✓ Stress results can be displayed on-screen
- ✓ Composite load effects available (by slicing)
- ✓ Construction cases readily created/ analysed
- ✓ Could use *tied mesh* for connection to slab



# Structural idealisation

## 5) Full 3D

- Although very powerful there is a tendency to make models overly complex too quickly
- Need to have an understanding of mesh refinement and FE principles

# Structural idealisation

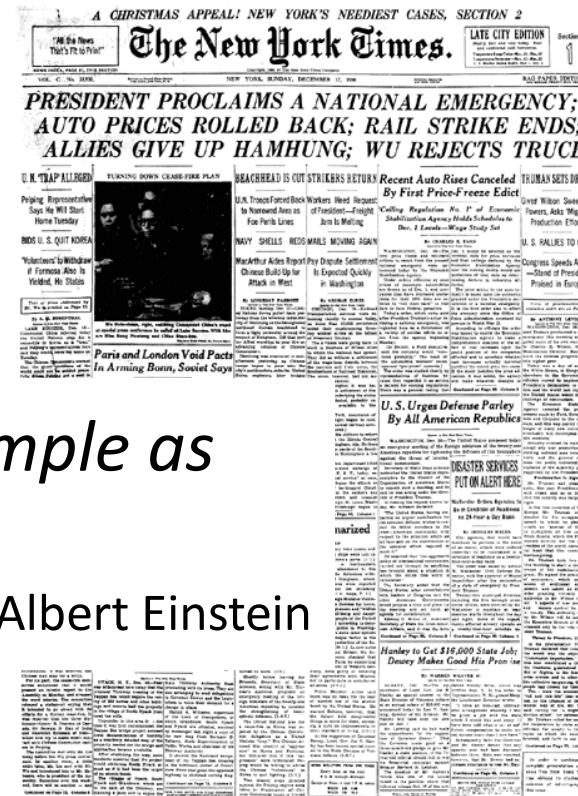
In principle:

- Represent the true behaviour of the structure to a level of accuracy which is *justified by the purpose of the analysis*
- “Deeper” analysis justified for
  - Large, costly structures
  - Existing structures. Assumptions acceptable for new build could lead to unnecessary disruption & waste of resources

# Structural idealisation

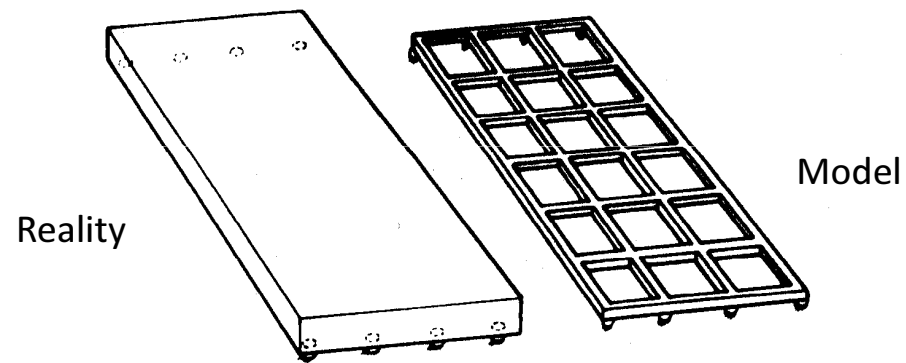
*Everything should be made as simple as possible, but not simpler*

Albert Einstein



# Introduction to grillages

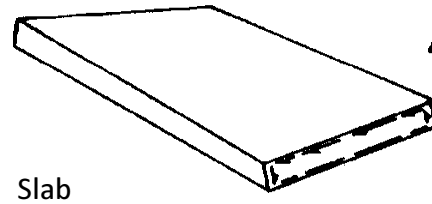
- The bridge deck is idealised as a grid of beam elements



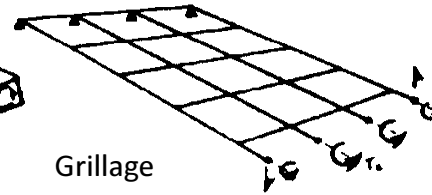
- Load distribution is all about relative stiffnesses
  - in this case, longitudinal vs transverse

# Introduction to grillages

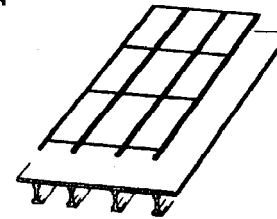
- Not mathematically rigorous but very rapid in solution & post-processing
- Track record for wide range of deck types



Slab



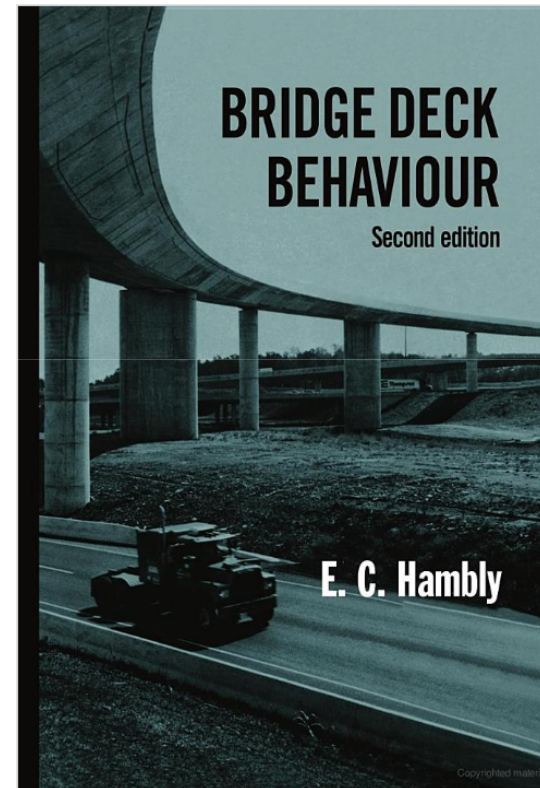
Grillage



Beam & slab decks

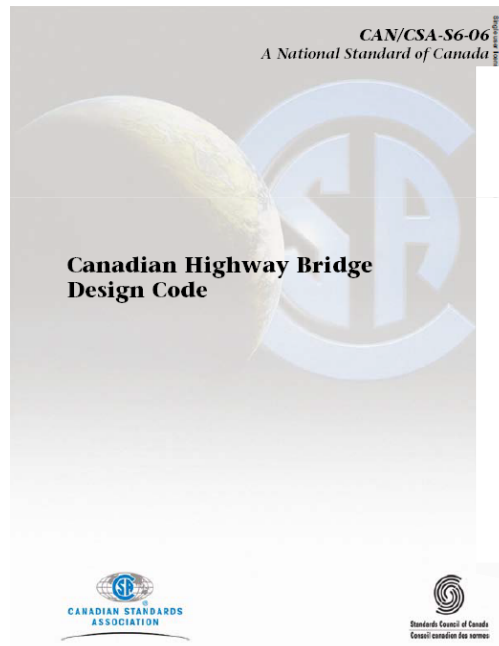
# Key references

- Hambly. Don't be without it!
- Gives practical rules:
  - Spacing of members
  - Torsional constants
  - Shear areas



# Key references

- Really good summary of Hambly.



**Table A5.2.2**

Bridge type and transverse section	Structural parameters				
	$i_L$	$i_T$	$j_L$	$j_T$	$s_v$
Composite slab-on-girder 	$\frac{I_x}{S}$	$\frac{t^3}{12(1-\nu^2)}$	$\frac{t^3}{6} + \frac{j}{S}$	$\frac{t^3}{6}$	May be ignored
where $I_x$ = the combined transformed moment of inertia of slab portion located in width $S$			where $j$ = transformed torsional inertia of the girder multiplied by $n_g = 0.88n$ for steel portions		
Multi-spine girder 	$\frac{I_x}{S}$	$\frac{t^3}{12(1-\nu^2)}$	$\frac{4A_e^2}{S \sum \frac{ds}{n_g t}}$	$\frac{t^3}{6}$	May be ignored for slab between spines. See note (*) for portion
where $I_x$ = the combined transformed moment of inertia of slab portion located in width $S$		for the portion of the deck between the spines. The value of $i$ for the portion of the deck included	where $n_g = 1.0$ for concrete portions		



# Grillage analogy

- Available freely from the web
- The first text to significantly add to Hambly in 40yrs



**NCHRP**  
REPORT 725

NATIONAL  
COOPERATIVE  
HIGHWAY  
RESEARCH  
PROGRAM

**Guidelines for Analysis Methods  
and Construction Engineering  
of Curved and Skewed  
Steel Girder Bridges**

TRANSPORTATION RESEARCH BOARD  
OF THE NATIONAL ACADEMIES



# Grillage analogy

**NCHRP**  
REPORT 725

NATIONAL  
COOPERATIVE  
HIGHWAY  
RESEARCH  
PROGRAM

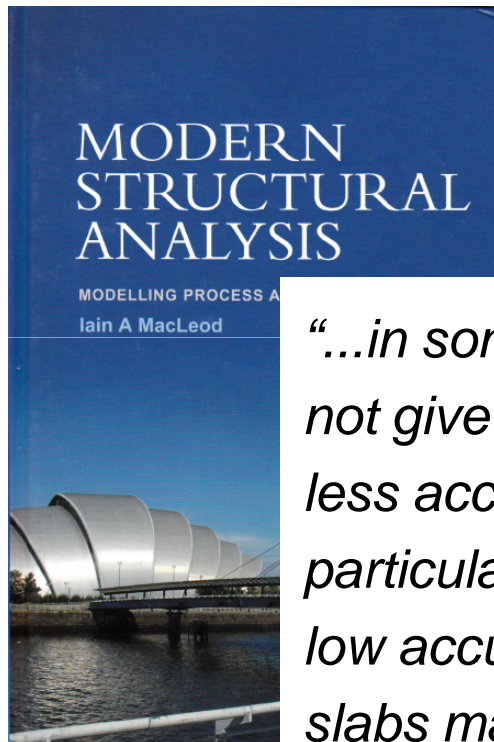
*“The conventional 2D-grid models used in current practice*

- *Substantially underestimate the girder torsional stiffnesses in I-girder bridges*
- *Substantially misrepresents the cross-frame responses*
- *Do not address the calculation of girder flange lateral bending in skewed I-girder bridges”*

Section 2.9



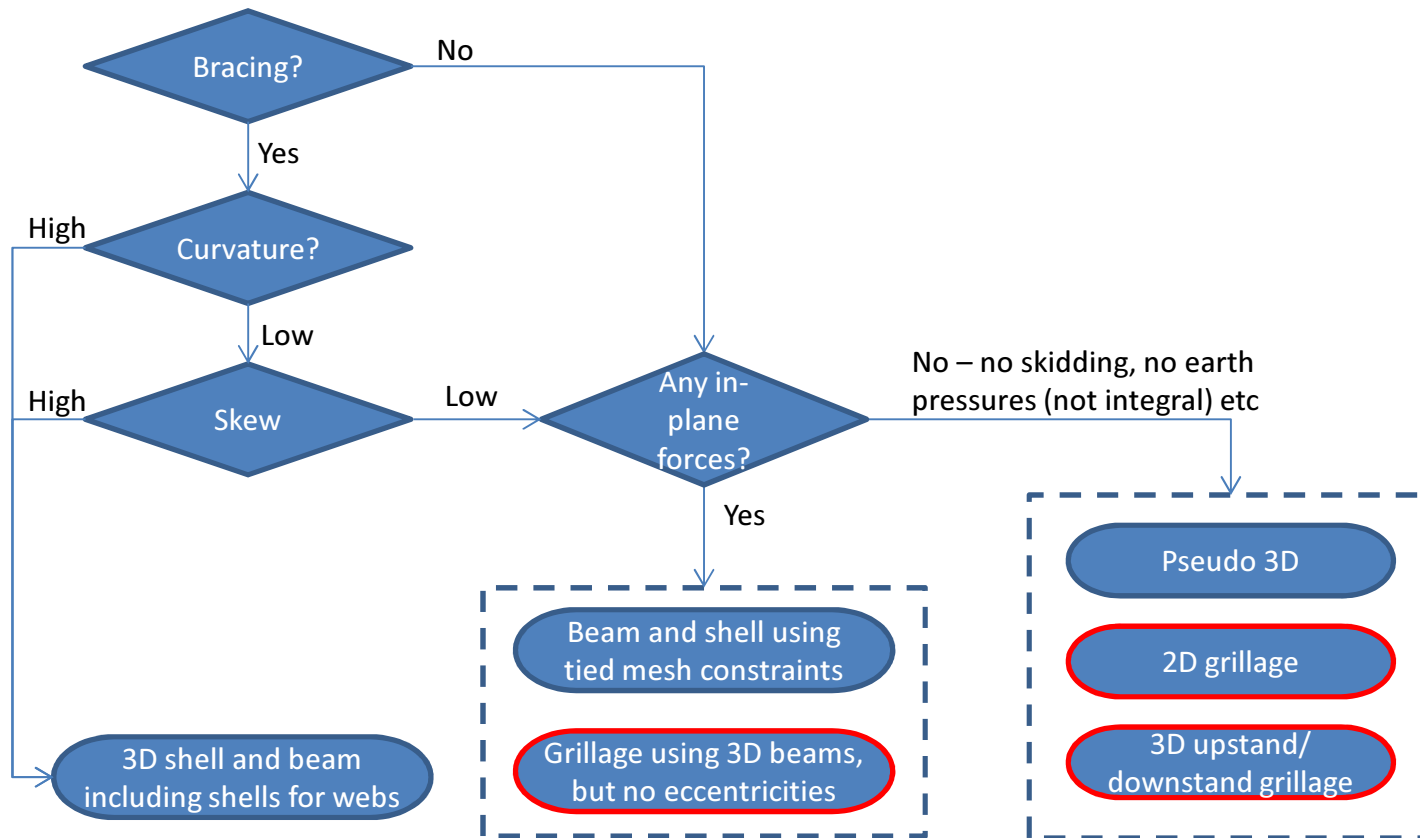
# Grillage analogy



*“...in some cases, refinement of grillage meshes does not give converging results and [can be] significantly less accurate than using plate bending elements. In particular, the prediction of twisting moments can be of low accuracy, hence the grillage modelling of skew slabs may give poor results.”*

Section 6.3.6

# When is a grillage model OK?



# Creating a grillage model

- Features: a grid of short lines
  - generally along beam centrelines
  - Ideally orthogonal, skews  $<20^\circ$  OK
  - Similar spacing longitudinal and transverse
- Grid spacing assumed
  - is equivalent to making an assumption about spreading elastic peaks over a width in an FE model
  - should be determined by rules from Hambly
  - may not improve accuracy if the spacing is reduced



# Grillage analogy

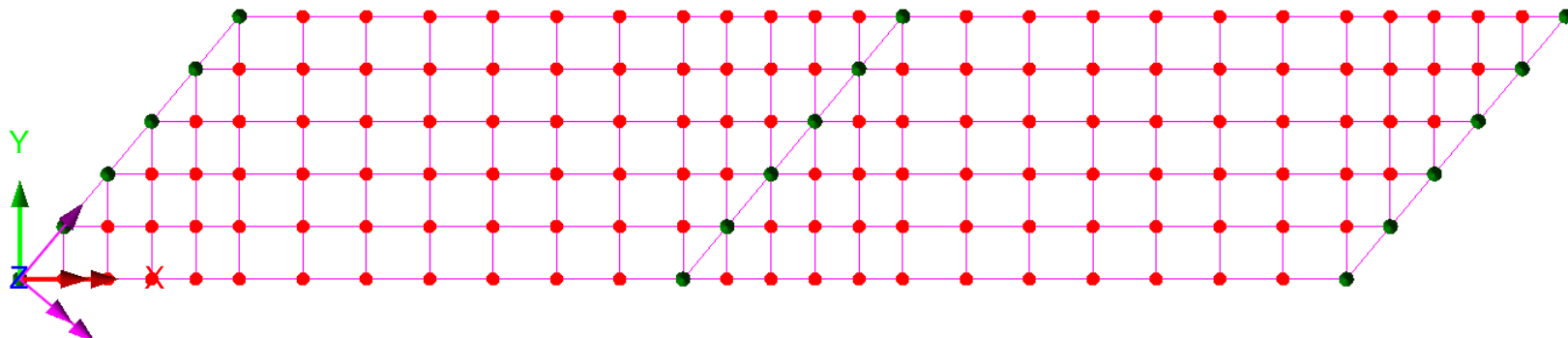
Deck Geometry

Do you require a grillage to represent a straight or curved deck?

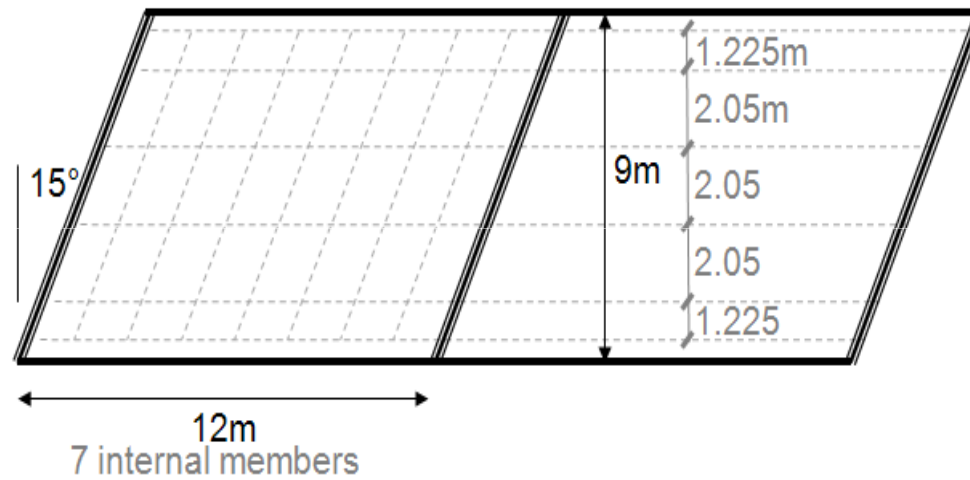
Straight  
Skew angle (degrees)

Curved  
Radius of curvature

 skew angle  span

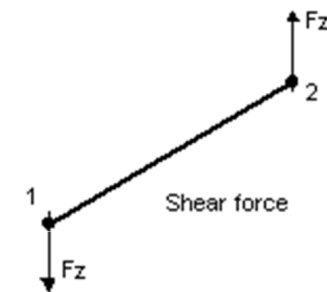
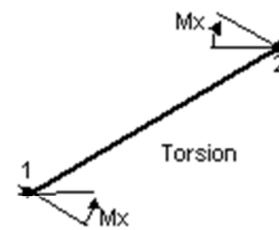
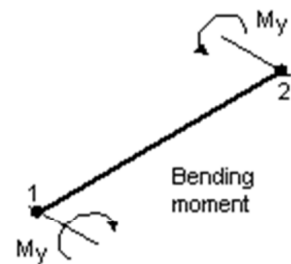


# Skewed Slab as Grillage Wizard demo



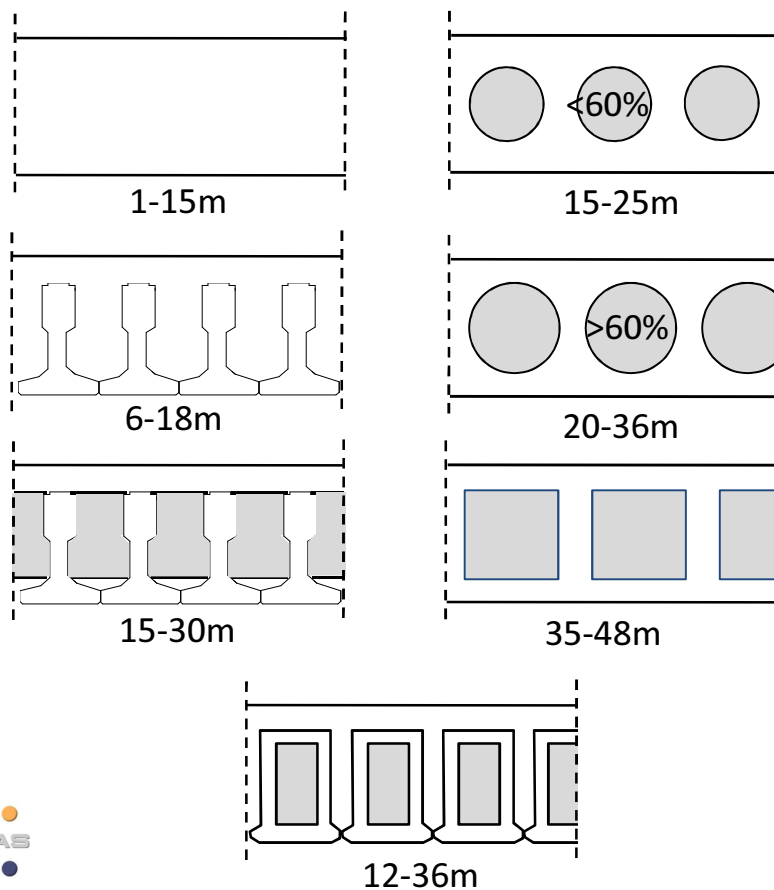
# Grillage analogy

- Grillage elements (2D) have 3DOF



- They do not carry axial load
- Grillages are for out-of-plane loads only i.e. vertical loads on bridge decks
- Usually 1 element on each side of a grillage bay. But can use more e.g. 2

# Flat slab deck types



- Isotropic slabs
  - Solid
  - Voids  $<60\%$  depth
- Orthotropic slabs
  - Beam & infill
  - Voids  $>60\%$  depth
- Multicellular
  - M-beams
  - Post-tensioned
- Shear key/ box beam



# Flat slab grillages

- Longitudinal members



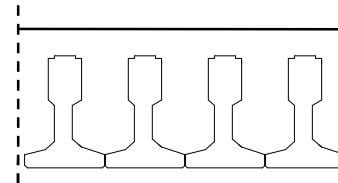
- must cross any discrete bearings
- spacing closer than  $0.25 \times \text{span}$  essential
- spacing closer than  $2d$  or  $3d$  pointless
- within  $0.3d$  of edge of slab

- Transverse members

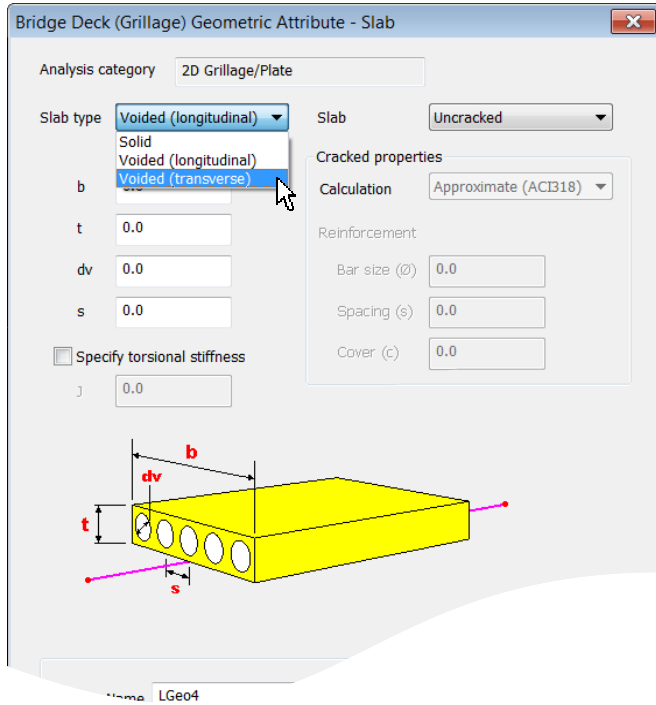
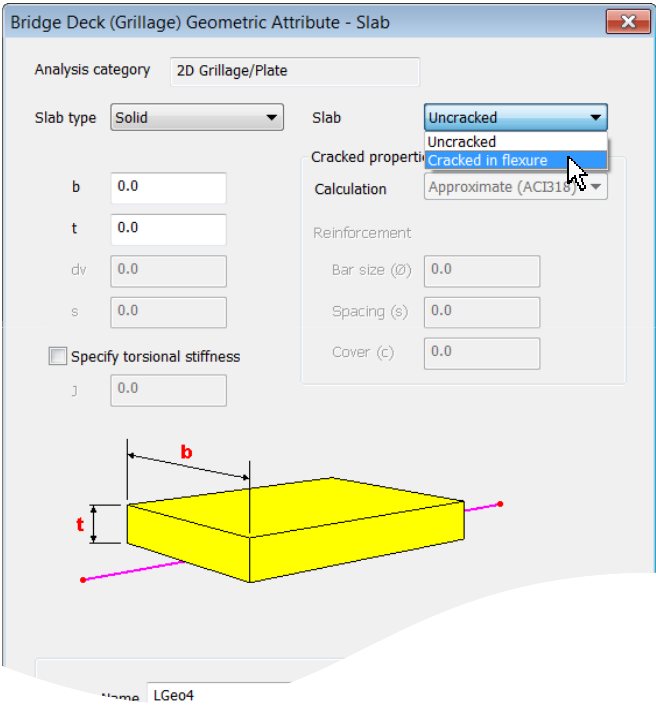
- Ideally orthogonal but skews  $< 20^\circ$  OK
- similar spacing to longitudinal

# Flat slab grillages

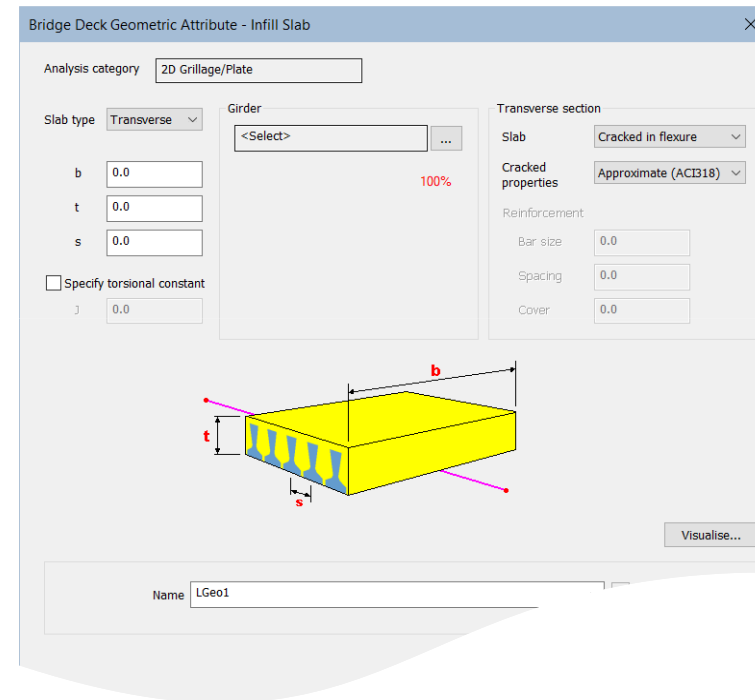
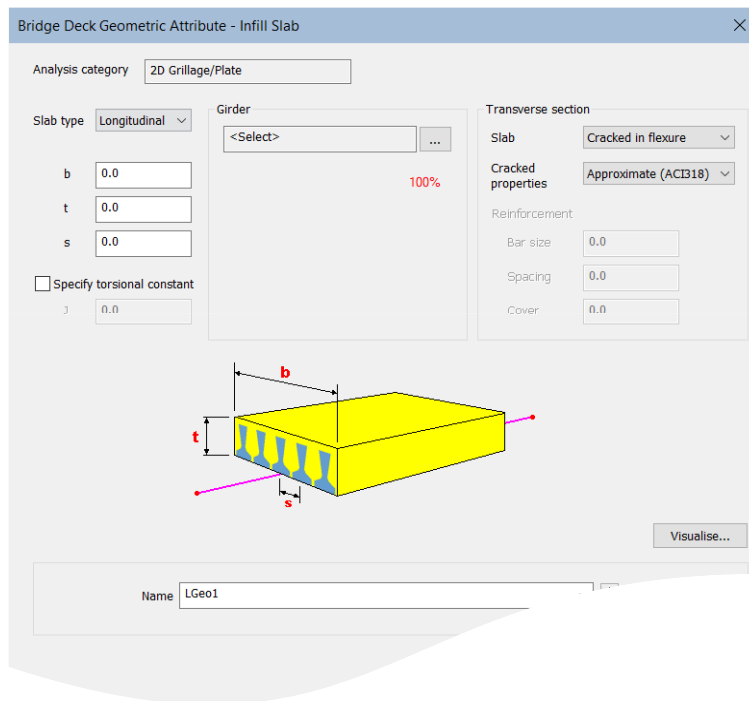
- PCC infill slabs
  - Dead load carried by bare beams so not included in loading on grillage
  - If longitudinal sections are prestressed, use gross section properties
  - If transverse sections are unprestressed (reinforced), use *cracked section properties*
  - Remember it's all about *relative stiffnesses*



# Grillage – slab



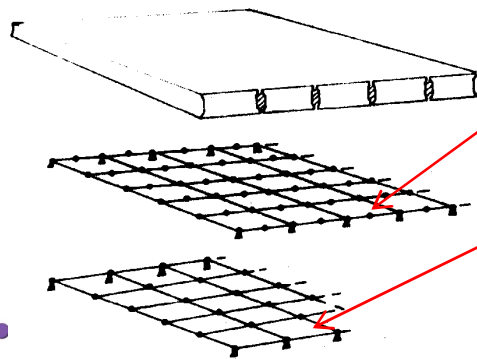
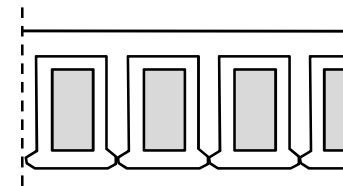
# Grillage – Infill



# Flat slab grillages

- Shear key decks

- Can use transverse mid-bay hinges
- Can use very short flexible members
- Else, transverse properties from Hambly (below)



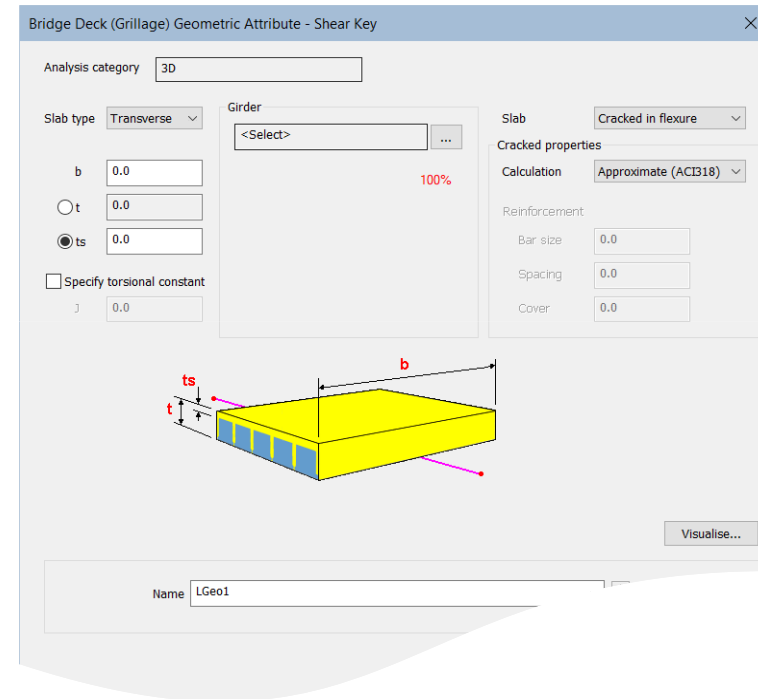
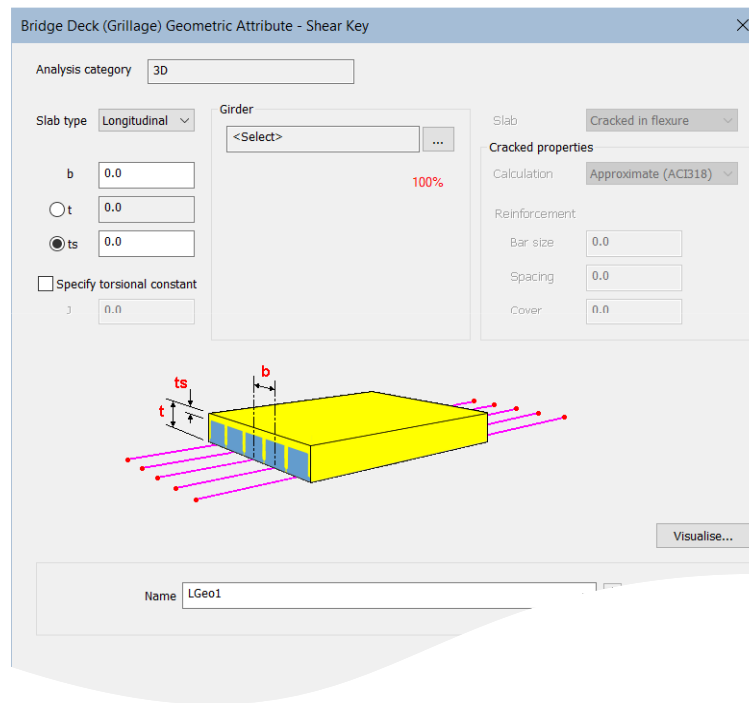
Possible mid-bay hinge or short flexible part

Alternative transverse member with the following properties:

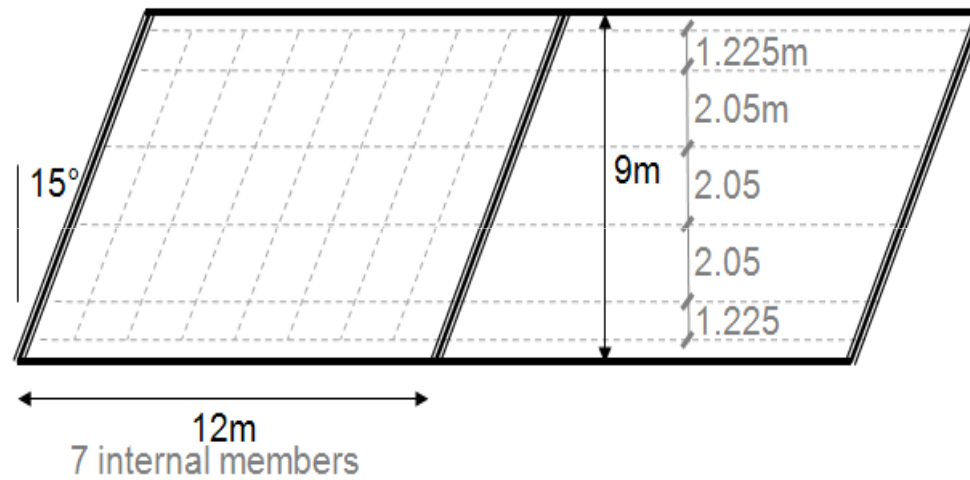
$$\frac{1}{I_e} = \left(1 - \frac{a}{l}\right) \frac{E_e}{E_1 I_1} + \frac{a}{l} \frac{E_e}{E_2 I_2}$$

$$\frac{1}{A_e} = \left[ \left(1 - \frac{a^3}{l^3}\right) \frac{1}{E_1 I_1} + \frac{a^3}{l^3} \frac{1}{E_2 I_2} - \frac{1}{E_e I_e} \right] \frac{l^2 G_e}{12}$$

# Grillage – Shear Key coming in Version 21



# Skewed Slab as Grillage Sections demo



# Results

- Results are generally per beam (not per unit width)
  - Can divide by width, where appropriate, using User Defined Results
- No stress concentrations
  - Up-front assumption for spacing eliminates
- Wood-Armer is only for isotropic slabs
  - And gives moments per unit width not per member





# Results

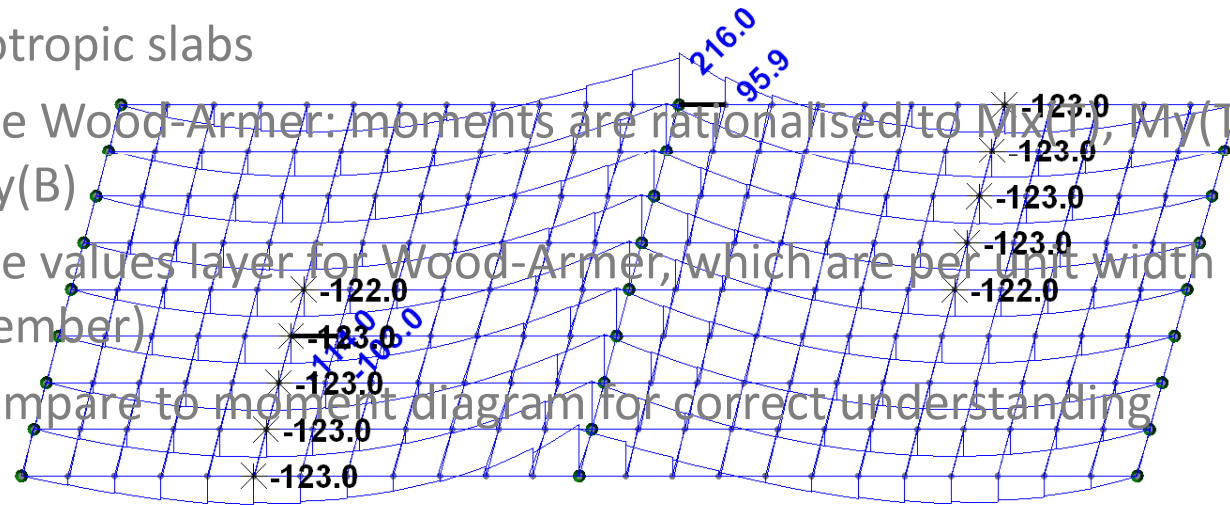
- Diagrams of stress resultants (M & V)

- Use groups to make less cluttered
- Isotropic slabs

- Use Wood-Armer: moments are rationalised to  $M_x(T)$ ,  $M_y(T)$ ,  $M_x(B)$ ,  $M_y(B)$

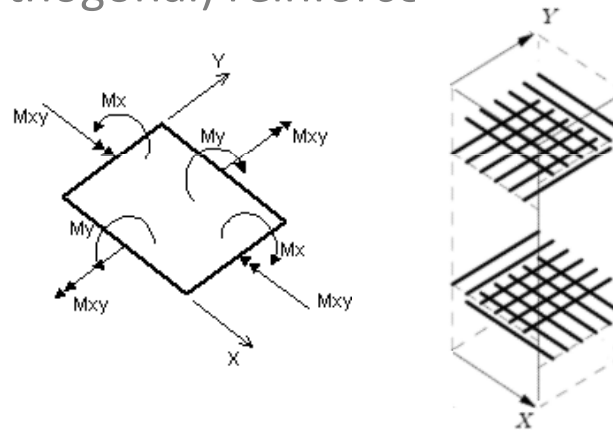
- Use values layer for Wood-Armer, which are per unit width (not per member)

- Compare to moment diagram for correct understanding



# Rationalising results

- Rationalise  $M_x$ ,  $M_y$  and  $M_{xy}$  for RC design
  - 4 layers of orthogonal (or near orthogonal) reinforcement
- Methods
  - Wood & Armer (1969)
  - Baumann (1972)
  - Stiglat (1983)

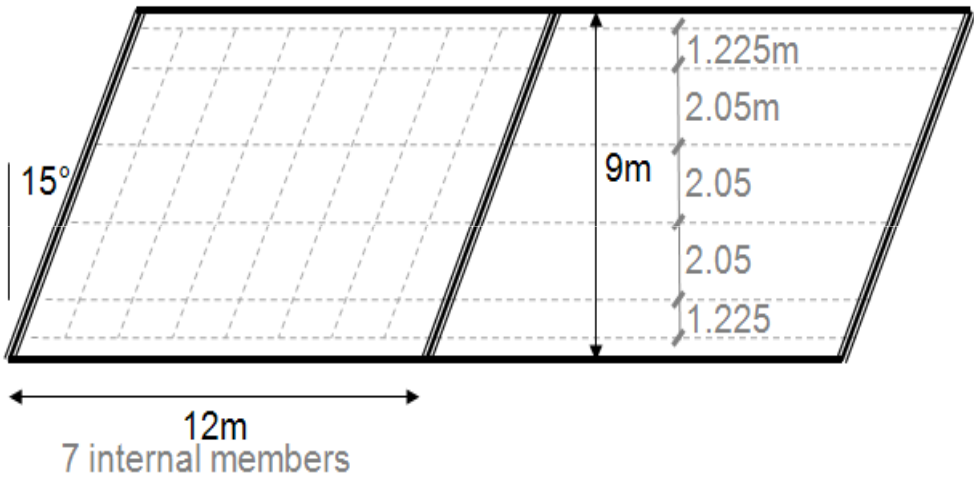


# Rationalising results

- Aim (in design) to optimise
  - i.e. Minimise total weight of reinforcement
- Wood Armer
  - Available in LUSAS
  - Design components:  $M_x(T)$ ,  $M_y(T)$ ,  $M_x(B)$ ,  $M_y(B)$
  - Assessment components (non-optimised): additional “moment of resistance” inputs required

The screenshot shows the 'Wood-Armer/Clark-Nielsen' dialog box. It is titled 'Wood-Armer/Clark-Nielsen' and has a close button (X) in the top right corner. The 'Analysis category' is set to '2D Grillage/Plate'. The 'Reinforcement angle' is set to '90.0'. There are two checkboxes: 'Wood-Armer' (checked) and 'Clark-Nielsen' (unchecked). A small coordinate system diagram shows the x and y axes. Below this, there is a section for 'Wood-Armer' design components. The 'Design components' section has two radio buttons: 'Minimised total weight/area of reinforcement' (selected) and 'k factor for non-minimised reinforcement' (with a value of 1.0). There is a checkbox for 'Display assessment utilisations'. Below this, there are two rows of input fields for 'Top rebar moment resistance' and 'Bottom rebar moment resistance', each with columns for 'x direction' and 'y' direction', all currently set to 0.0. A diagram of a reinforced concrete slab cross-section is shown, with labels for 'dx' and 'dy' at the top and bottom edges. To the right of the diagram are input fields for 'dx' and 'dy' for 'Top' and 'Bottom' reinforcement, all set to 0.0. At the bottom, there is a 'Name' field containing 'WdAmr1' and a '(new)' button. The dialog box has 'OK', 'Cancel', 'Apply', and 'Help' buttons at the bottom.

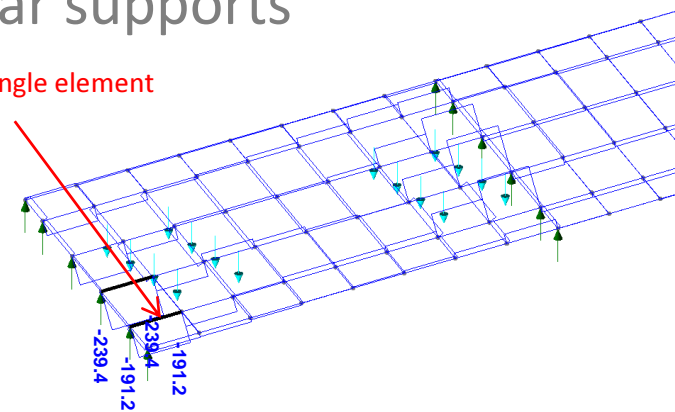
# Skewed Slab Wood-Armer demo



# Results – caution

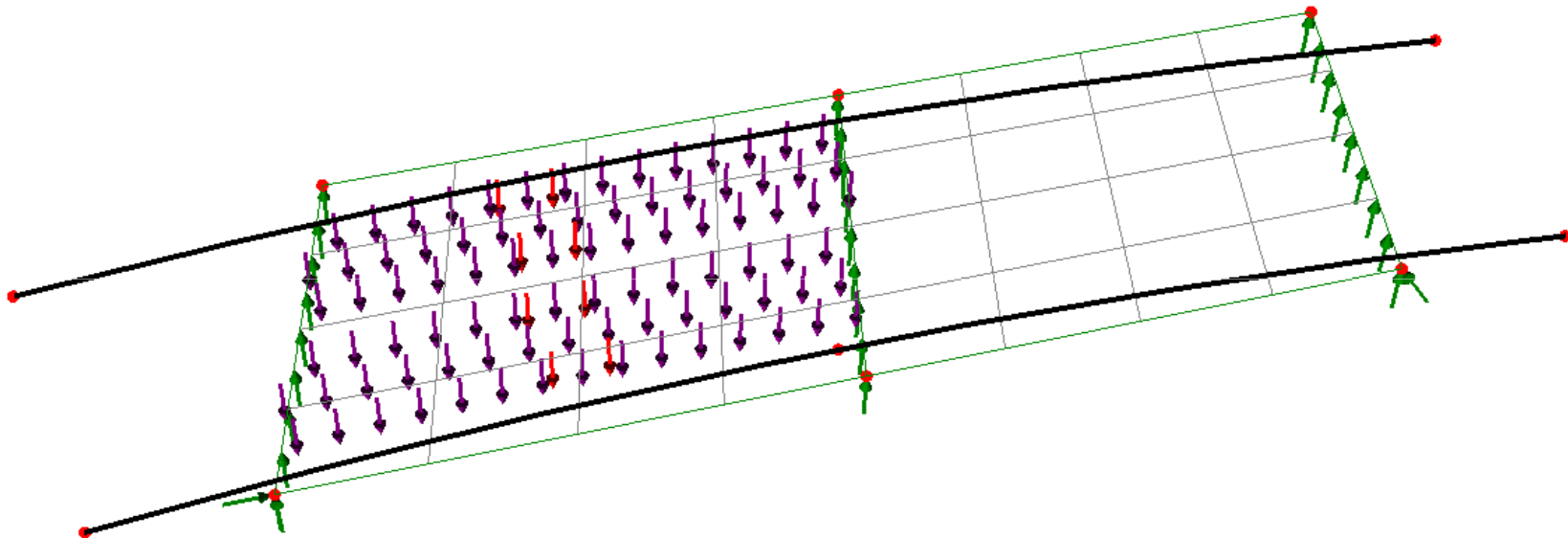
- Misreporting of shear near supports

No change in shear over a single element



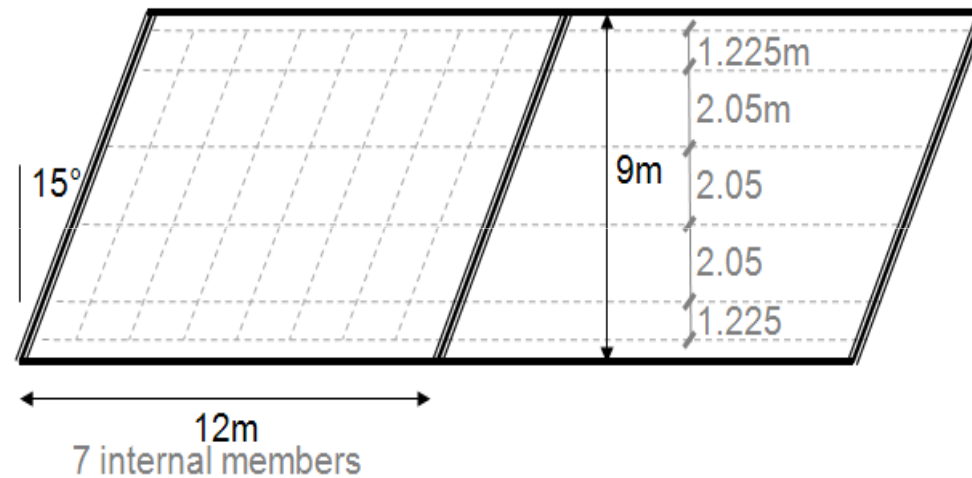
- Caused by distribution of loads to nodes
- Inherent to grillage analysis (not LUSAS-specific)

# Influence analyses & VLO demo



# Simple Skewed Slab as Grillage

- Homework example is the same model as shown in today's session.



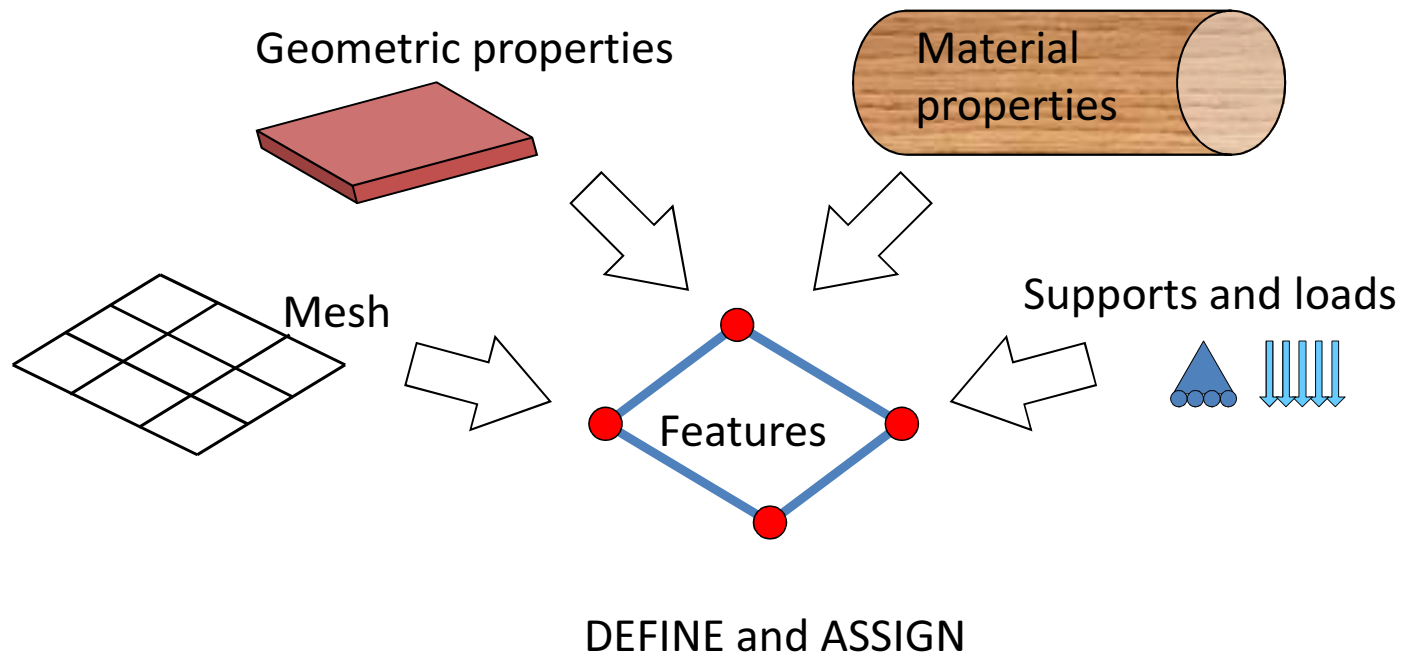
# Worked Example

- Complete Simple grillage example
- Questions on the module, example or quiz can be sent to:
  - [onlinetraining@lusas.com](mailto:onlinetraining@lusas.com)





# LUSAS Model Attributes



# Basic checklist

- Reactions
- Deformed shape
- Magnitude of deformations
- Warning or error messages
- Mesh refinement

# Worked Example and Quiz

- Complete Skewed Slab as grillage example
- Session 1 Quiz, on the module and Slab example at:
  - [https://www.lusas.com/grillage\\_oct\\_23/index.html](https://www.lusas.com/grillage_oct_23/index.html)  
([https://www.lusas.com/grillage\\_oct\\_23/index.html](https://www.lusas.com/grillage_oct_23/index.html))
  - Username: **session1**
  - Password: **LazyDog77##**
- Questions on the module, arch example or quiz can be sent to:
  - [onlinetraining@lusas.com](mailto:onlinetraining@lusas.com)



# Next Session

- **Session 2 – Advanced Grillage Analysis**
  - Building on session one, more complex bridge structures, such as multi-span composite bridges and box bridges will be discussed, covering how they can be modelled.
- 19<sup>th</sup> October (next week) at 14:00 GMT (same time as today's session)
- **Please register!**





Online training course

Thank you for attending the session

Contact: [onlinetraining@lusas.com](mailto:onlinetraining@lusas.com)