

CUSTOMER SUPPORT NOTE

Nonlinear Buckling Analysis with Initial Imperfection

Note Number:	CSN/LUSAS/1014
--------------	-----------------------

This support note is issued as a guideline only.



Forge House, 66 High Street, Kingston upon Thames, Surrey, KT1 1HN, UK
Tel: +44 (0)20 8541 1999 Fax: +44 (0)20 8549 9399
Email: info@lusas.com www.lusas.com

Table of Contents

1.	INTRODUCTION	1
2.	DESCRIPTION	2
3.	IS A NONLINEAR BUCKLING ANALYSIS NECESSARY?	3
4.	HOW MUCH INITIAL IMPERFECTION SHOULD BE USED?	4
5.	EXAMPLE	5
6.	NOTES ON NEGATIVE PIVOTS AND LOCAL BUCKLING	10
7.	REFERENCES	12

1. Introduction

Following the review of results from an Eigenvalue Buckling Analysis, the analyst will consider whether or not a Nonlinear Buckling Analysis is required. There are various nonlinearities that may be considered for inclusion. However, even if only elastic buckling is being considered and there is no contact or boundary condition nonlinearity, a Nonlinear Buckling Analysis may be needed to consider second order (geometric nonlinearity (GNL)) effects. This may indeed also be a requirement of the design code the engineer is working to.

In a nonlinear buckling analysis the buckling load factor can be determined as the load factor reached for the first converged increment (not iterations) where a negative pivot occurs. This can be established by opening the output file (file extension .out) or in the log file (file extension .log) in a text editor such as NotePad. The latter file is an echo of the Solver window which is seen on the screen whilst the model is solving. The first negative pivot will be indicated by the parameter "**NSCH**" = 1. At this point the solution may struggle, or fail to converge further on a post-buckling path. Once Solver converges on a post-buckling path the negative pivots should disappear as the model becomes stable again (this is an indication that the negative pivots did indeed indicate buckling and not another instability).

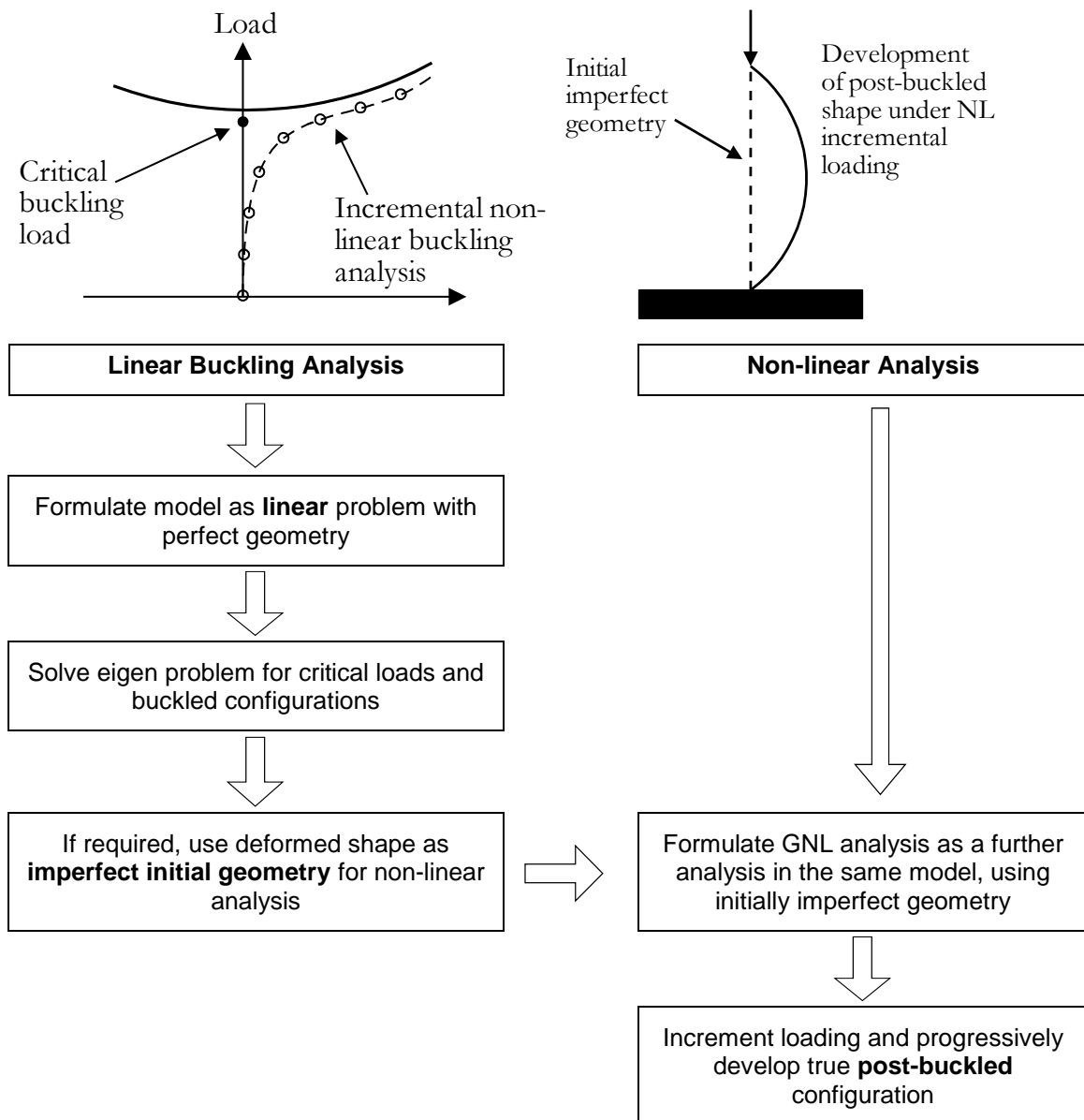
If a post-buckling solution is required, then a small perturbation (trigger) load could be applied to encourage the solution onto a particular post-buckling path. This load should be small enough such that it does not affect the results, but just disturbs the symmetry of the mesh enough to encourage a particular post-buckling path.

An alternative method for a post-buckling solution is to use slightly deformed initial geometry. This may be achieved when defining the geometry, or by using the deformed mesh from a separate eigenvalue buckling analysis. This mode shape may be used as a starting point for the mesh in a subsequent and separate nonlinear buckling analysis. This latter method is the subject of this Support Note and is often the most suitable and convenient method where an Eigenvalue buckling analysis has already been performed in the same model.

2. Description

To achieve a post-buckling solution, an initial deformed shape can be either created by actually building the model with an imperfection in the geometry or by using the deformed shape from another analysis (say from eigenvalue, static or linear eigenvalue buckling analysis).

A linear (eigenvalue) buckling analysis will produce critical buckling loads and buckled shapes. The results of this analysis are normalised displacements due to eigenvector unity normalisation. A factored deformed shape from this linear buckling analysis can then be used as the initial mesh for a further nonlinear buckling analysis to develop post-buckling behaviour.



3. Is a Nonlinear Buckling Analysis Necessary?

Imperfections and nonlinearities tend to prevent most engineering structures from reaching their theoretical elastic (or "Euler") buckling strength. Eigenvalue buckling load factors are therefore usually overestimated - the results should be regarded as unconservative, as they most often are.

Design codes provide empirical calculations to determine critical buckling factors and suitable stiffeners, but these may lead to over conservative design. Some design codes allow the designer to use a second-order analysis to determine the buckling load factor more accurately and this can create greater economy of design. There is a case study by Atkins on our website http://www.lusas.com/case/bridge/critical_buckling_analysis.html comparing buckling load factors determined by different methods and analyses for a particular structure. Chris Hendy is quoted as saying:

"Not only did the nonlinear LUSAS model give results almost identical to the actual physical test specimen but it also showed that the EN 1993-1-5 rules for stiffeners were very conservative for this particular beam, and the BS 5400 Part 3 predictions even more so."

Eurocode 4 [B2] cl.5.2.1(2) states that second-order effects should be included if they cause significant increase of the action effects, or significant change to the structural behaviour. Then cl.5.2.1(3) states that the results from a first-order analysis can be used if the effect of the deformation (second-order effects) will increase the internal forces and moments by less than 10%. To determine this with confidence a second-order analysis is required. However, the clause goes on to say that this can be assumed to be the case if the elastic critical buckling load factor is greater than 10.

In other words, if you are working to Eurocodes and have run a first-order Eigenvalue Buckling Analysis and you have determined a buckling load factor less than 10, then second-order effects must be considered and a Nonlinear Buckling Analysis performed.

The NCHRP 725 [N1] section 3.3.3.3 gives the following formula (eq2) for an amplification factor AF_G :

$$AF_G = \frac{1}{1 - \frac{M_{maxG}}{M_{crG}}}$$

Where:

M_{maxG} is the maximum global moment applied

M_{crG} is the critical elastic buckling moment – from the simple formula or from an Eigenvalue buckling analysis.

If the amplification factor AF_G is less than 1.1 then the guidance in the report is that second-order effects can be neglected (similar limit to that mentioned in the Eurocodes). If the amplification factor is greater than 1.25, the report recommends using a second-order (or "geometric nonlinear") 3D FE analysis.

4. How much initial imperfection should be used?

For a nonlinear buckling analysis where you are using an initial imperfection, from a mathematical point of view, the minimal imperfection required should be used. This is to avoid reducing the buckling load determined, but at the same time to be just enough to encourage solution past buckling and onto a particular post-buckling path. For example an imperfection equal to the span length, $L/1000$ would usually suffice. However, for the purposes of design, a larger imperfection will be more conservative and an initial imperfection in the order of $L/300$ may be a good starting point for bridge beams.

EN1993-1-1 [B1] suggests using the shape of the elastic critical buckling mode as an imperfection when second order analysis is used (see clause 5.3.4) with the amplitude based on the section in question (see Table 6.2 and Table 5.1 in conjunction). Broadly speaking, the imperfections are of order $L/150$, or $L/300$ for heavy bridge sections if LTB is concerned.

BS5400-3 [B3] clause 9.12.1 recommends that initial imperfections for use in nonlinear buckling analyses should be 1.5 times the relevant tolerances given in BS5400-6:1999 [B4] Table 8 - of order $L/667$ or 5mm (whichever is greater) for a bridge girder.

AASHTO [A1] doesn't appear to cover imperfections for second order buckling analyses. However, if we look to AISC 303-10 [A2], this gives fabrication tolerances (section 6.4) which (for spans >30 ft) come down to $L/1000$ or $1/8$ ", whichever is greater, and erection tolerances which, in broad terms come down to $L/500$ (section 7.13.1.2). As stated above, BS5400-3 [B3] suggested that assumed imperfections should be $1.5 \times \text{tolerance}$, and since AASHTO and AISC are silent, the 1.5 factor seems reasonable - we find ourselves around $L/300$, as for the Eurocode.

5. Example

This example picks up from the end of the eigenvalue buckling analysis of the plate girder; “**Buckling Analysis of a Plate Girder**” in the **Application Examples Manual (Bridge, Civil and Structural)** after it has been run and the buckling load has been calculated for the girder. In Figure 1 the deformed shape of the girder is shown.

When an eigenvalue buckling analysis is performed the displacements are unity normalised, in this case as the model has been built in metres unity being one metre.

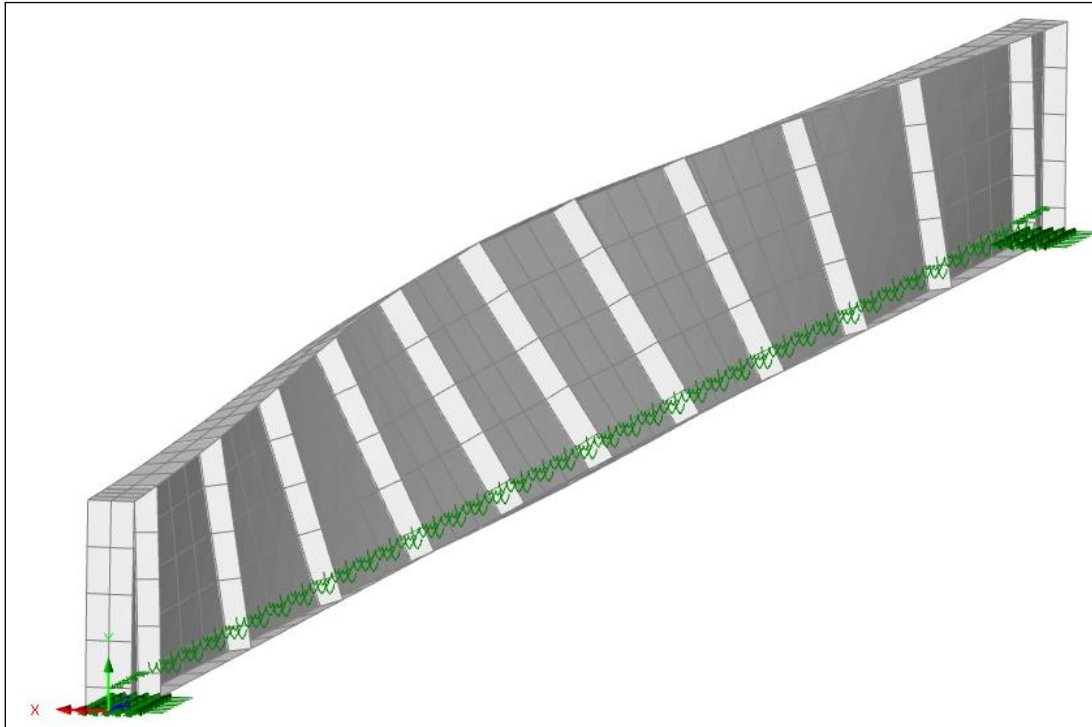


Figure 1

To create a Nonlinear Buckling Analysis in the same model using an initial imperfection from the deformed mesh of the first buckling model of a prior Eigenvalue Buckling Analysis, the required steps are as follows:

1. Create a new analysis in the same model for a subsequent nonlinear buckling analysis. This can be done via the **Analyses** menu by choosing to create a new **General Structural Analysis**.
2. Inherit **All** assignments from the Eigenvalue Buckling Analysis (Analysis 1) as shown in Figure 2 below:

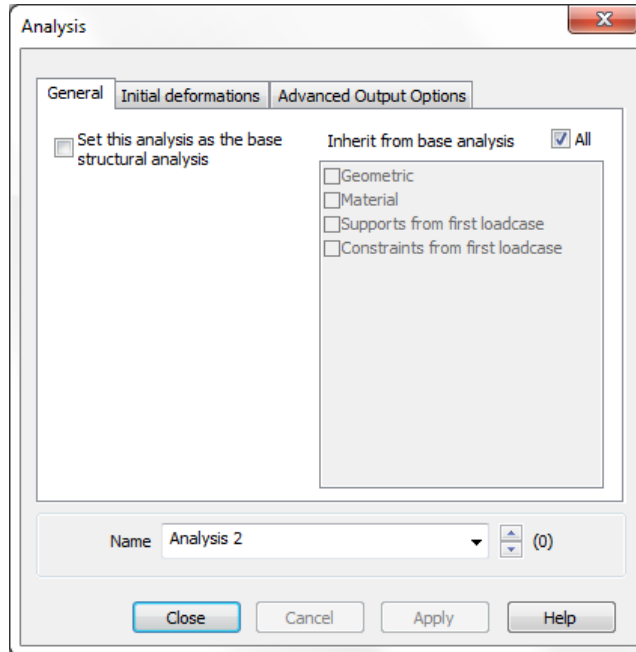


Figure 2

3. On the **Initial Deformations** tab choose to "**Start with deformed mesh from**" and Analysis 1 and specify the loadcase and the particular eigenvalue result to use (normally the first buckling mode "1"). This is shown in Figure 3 below.

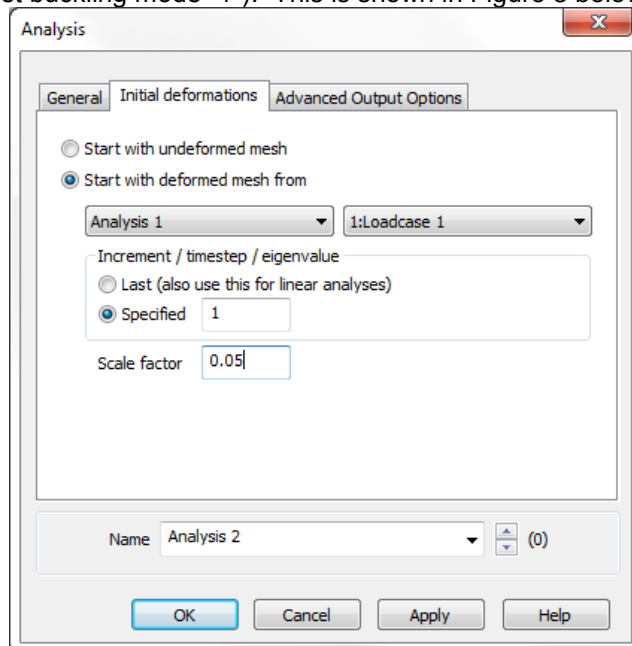


Figure 3

Enter the **Scale factor** that is required. Entering a value of 0.05 in this case scale the one metre deflection down to 50mm. This degree of imperfection is based on the span length divided by 300, which is a considerably larger imperfection than the expected fabrication tolerance, including therefore some allowance for any residual stresses. The design code that is being worked to should be referred to for guidance with regard to the amount of imperfection to use that should be used (see section 3 above).

4. Click **OK**.
5. Copy the "**Loading**" from the Eigenvalue Buckling Analysis (Analysis 1) into the new Nonlinear Buckling Analysis (Analysis 2). You can do this in the Analyses Tree View by holding **Ctrl** on the keyboard (to copy rather than move) and then dragging and dropping the "**Loading**" folder under Analysis 1 onto Analysis 2 as shown below in Figure 4:

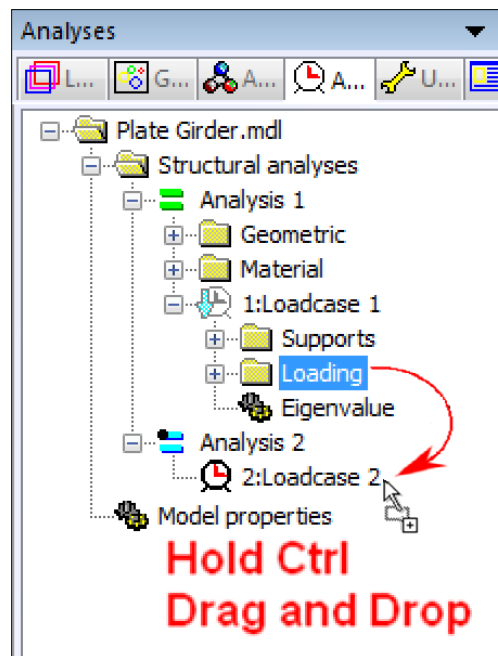


Figure 4

6. Delete the **Eigenvalue** control from the new, Analysis 2 loadcase, "2: Loadcase 2"
7. In this new analysis, to specify a nonlinear solution, you must add a **Nonlinear and Transient Control** to the Loadcase (right-click on the loadcase>**Controls**>**Nonlinear and Transient**). Example settings that can be used this example are shown in Figure 5 below.

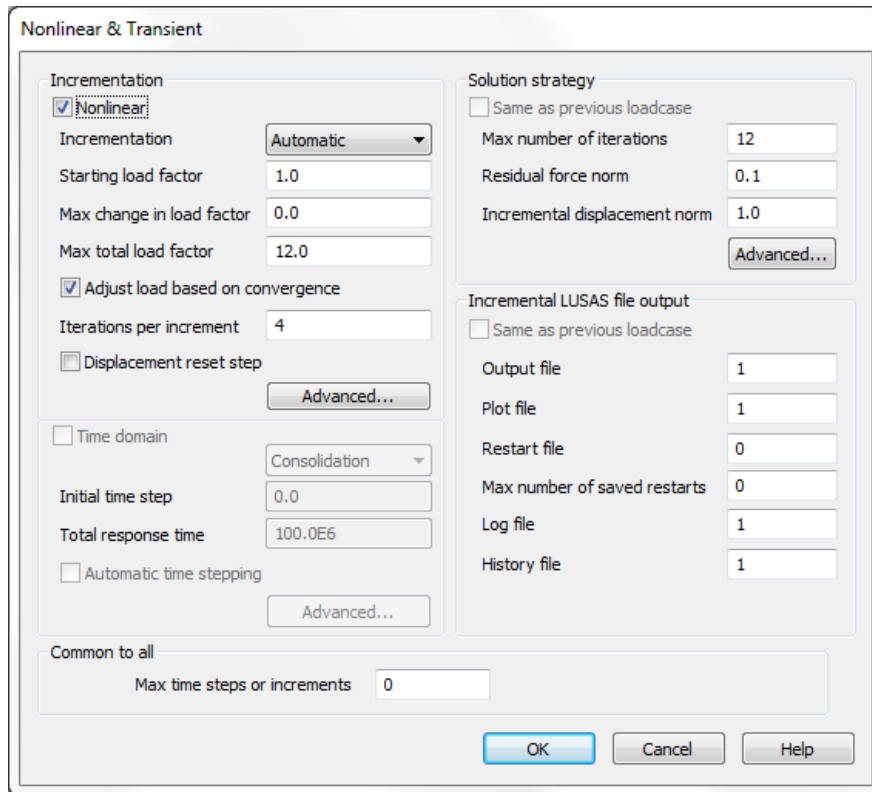


Figure 5

8. The inclusion of Geometric Nonlinearity (GNL) is required in a nonlinear buckling analysis. For a Nonlinear Buckling Analysis, elements that have a geometric nonlinearity formulation must therefore be used. The geometric nonlinearity formulation supported by the element type(s) being used can be checked in the **Element Reference Manual**. The chosen suitable formulation to be used is then set in the **Nonlinear Options** in the Analyses TreeView (or via **File>Model Properties>Solution tab>Nonlinear Options**) as shown in Figure 6 below. If different elements are used with different formulations for GNL, more than one option can be selected in the **Nonlinear Options**.

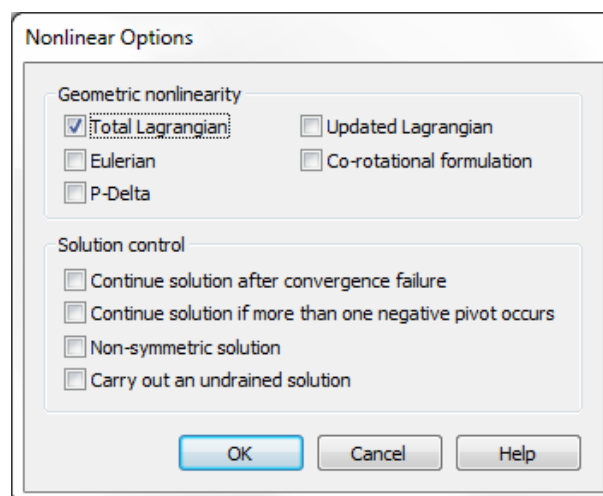


Figure 6

In this case the **Total Lagrangian** option has been selected as this is suitable for use with the **Semiloof Thin Shell** elements (**QSL8**) used in the example. The **Element**

Summary Table for shell elements in the **Element Reference Manual** is shown below with the two suitable formulations for QLS8 identified:

		Plates			Shells						Mem			
		TF3, QF4	QSC4	TTF6, QTF8	BXS3	TS3, QSI4	TSR6	TSL6, QSL8	TTS3	TTS6	QTS4	QTS8	BXM2/3	TSM3, SMI4
Product Version	LT, Standard (S) or Plus (+)	S	S	S	S	S	+	+	S	+	S	+	S	S
Nonlinear	Total Lagrangian				✓		✓	✓	✓	✓	✓	✓	✓	
	Updated Lagrangian				✓		✓	✓						
Geometry	Eulerian													
	Co-rotational						✓							
Integration	Explicitly Integrated													
	Numerically Integrated	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Mass	Consistent Mass (default)	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
	Lumped Mass	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Figure 7

9. Solve the Nonlinear Buckling Analysis.

A graph could then be plotted using **Graph Wizard (Utilities menu)** to show the nonlinear response due to buckling. A plot of the total load factor against the lateral displacement (DX) of a node on the top flange at mid-span would be suitable for looking at lateral torsional buckling.

The method described here adjusts the coordinates of all nodes in the mesh for the Nonlinear Buckling Analysis only and the updated node coordinates will be tabulated for this analysis to the Solver Data file to include the imperfection using the supplied deformed mesh factor.

Please see the **Modeller Reference Manual>Chapter 7 - Running an Analysis>LUSAS Analyses Types>Nonlinear Analysis** and the guidance in the user area of our website for nonlinear analyses in general.

Note:

The GNL analysis may reduce the total load factor when compared with the Eigenvalue buckling solution, but it's still considered unsafe, as the material is assumed elastic here. Stresses can be checked to see if they exceed yield at each increment solved and so it can be checked whether this occurs before or after buckling. Material nonlinearity should also be included in the nonlinear analysis to consider what effect yielding has on later buckling or vice versa. Yielding can of course considerably reduce the Total Load Factor determined, so it is important that it is not overlooked.

6. Notes on Negative Pivots and Local Buckling

The following points should be noted when performing nonlinear buckling analyses:

1. Local buckling modes occurring for lower load factors than a global buckling mode in an Eigenvalue buckling analysis should be considered carefully and not simply 'filtered out' or ignored in haste.
 - i. Local modes can lead to a more global collapse. For example in a truss bridge there tends to be little redundancy and buckling of one member could lead to a chain reaction that ultimately results in collapse.
 - ii. Apparent local modes in an Eigenvalue analysis can also turn out to also be a global mode as well. To check this, make the local member that is buckling with more obvious displacement invisible and check deformation of the rest of the structure in general.
2. A negative pivot will occur if an individual member in the model reaches a buckling load, or if a global buckling load is reached.
3. An imperfection can be used to allow Solver to get past a negative pivot, and the negative pivot is cleared and then a stable, post-buckling solution is returned for further converged increments.
4. If an imperfection for a member is large enough, no negative pivot will be seen in any iteration even though the buckling load may have been exceeded and a post-buckling solution is returned.
5. If an imperfection for a member is too small a negative pivot may occur, but may remain for further solved increments with Solver allowing one negative pivot for a converged increment, despite an unstable solution being returned with NSCH non-zero for the converged increments
6. If an imperfection for a member is just sufficient, then a negative pivot may occur in an iteration of an increment and then may clear for a later iteration with a post-buckling solution obtained. How sensitive the occurrence of a negative pivot is to the magnitude of imperfection is expected to vary between models.
7. A very small imperfection for a member may even encourage a post-buckling solution for that member buckling the opposite way.
8. The smaller the imperfection, whilst still sufficient, the more prominent the buckling load on a graph of force vs lateral displacement due to a more abrupt change in gradient (stiffness).
9. A stable post-buckling solution can still be obtained without an imperfection if the general deformation of the loaded structure leads to out-of-straightness in a member that then buckles in compression. For such case, a negative pivot may or may not occur for an iteration.
10. An imperfection based on a global buckling mode may or may not also provide sufficient imperfection to local members for a post-buckling solution for local buckling to be obtained. If it does not, negative pivots will be obtained.
11. More than one negative pivot can be returned for the same local buckling of one member. This means that the occurrence of a second negative pivot cannot be assumed to be a second local buckling mode, nor can it be assumed to indicate global buckling.
12. Solver will still converge and give results if there is only one negative pivot, even though it is for an unstable solution for having loaded past either a local or global buckling mode. Users must check the output file for negative pivot warnings for converged increments.
13. Solver will still converge and give results for the first increment that has two negative pivots, but errors and fails when about to begin the next increment.

14. It is therefore recommended that the **Graph Wizard (Utilities menu)** is used to plot the **"Number of negative pivots"** for a converged increment against the **"Increment number"** to quickly check whether any increments have given a solution with a negative pivot. This is a **"Time History"** plot using the **"Named"** option for **Entity Data**.

7. References

- A1: AASHTO LRFD “*Bridge Design Specifications 6th Edition*”, American Association of State Highway and Transportation Officials, Washington, 2012
- A2: AISC 303-10 “*Code of Standard Practice for Steel Buildings and Bridges*”, American Institute of Steel Construction, Chicago, April 2010
- B1: BS EN 1993-1-1:2005, “*Eurocode 3: Design of steel structures – Part 1-1: General rules and rules for buildings*”, British Standards Institute, London, Sept 2006
- B2: BS EN 1994-1-1:2004, “*Eurocode 4: Design of composite steel and concrete structures – Part 1-1: General rules and rules for buildings*”, British Standards Institute, London, Feb 2005.
- B3: BS 5400-3:2000, “*Steel, concrete and composite bridges: Part 3: Code of practice for design of steel bridges*”, British Standards Institute, London, May 2001.
- B4: BS 5400-6:1999 “*Steel, concrete and composite bridges - Part 6: Specification for materials and workmanship, steel*”, British Standards Institute, London, Oct 2002
- N1: NCHRP (National Cooperative Highway Research Program) Report 725 “*Guidelines for Analysis Methods and Construction Engineering of Curved and Skewed Steel Girder Bridges*”, Transportation Research Board, Washington DC, 2012