CUSTOMER SUPPORT NOTE

Buckling Analyses

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This support note is issued as a guideline only.

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1. Introduction

A linear static analysis is suitable for structures that are stiff and where stress levels are below the yield or cracking limits. It can also be used when geometric and material nonlinearities are important, with results from the linear elastic analysis applied to simplified methods in codes that consider these effects. However, for slender structures or when more detailed analysis is needed, advanced methods may be required. This support note explains the methods available in LUSAS for analysing a structure's buckling behaviour. The three main approaches are:

- Linear Eigenvalue Buckling Analysis
- Geometrically Nonlinear Analysis
- Geometrically and Materially Nonlinear Analysis.

These methods are described in more detail below. In Geometrically Nonlinear Analyses and Geometrically and Materially Nonlinear Analyses, boundary condition nonlinearity can also be taken into account, if necessary.

2. Linear (Eigenvalue) Buckling Analysis

2.1 General information

An eigenvalue buckling analysis provides buckling load factors and their corresponding mode shapes. Buckling load factors are the factors by which applied loads must be multiplied to cause buckling. An eigenvalue buckling analysis assumes linear elastic behaviour and, as a result, typically provides an upper-bound estimate of the structure's buckling load. However, when a structure is relatively stiff, and geometric and material nonlinear effects are negligible, the computed buckling load may closely approximate the actual load at which buckling occurs. It is also worth mentioning that an eigenvalue buckling analysis does not provide post-buckling information; a nonlinear buckling analysis is required to capture post-buckling behaviour.

This type of analysis identifies both local and global buckling modes. However, engineering judgement is essential to determine the most critical mode and select the appropriate buckling load factor. The resulting modes can also be visually inspected in Modeller to assist in this evaluation.

The fundamental assumptions of an eigenvalue buckling analysis are as follows:

- The linear stiffness matrix does not change before buckling occurs.
- The stress stiffness matrix is simply a scaled version of the initial stiffness matrix.

Since the analysis requires the use of a stress stiffness matrix, it follows a nonlinear solution path in LUSAS. Therefore, linear eigenvalue buckling analysis is only available for elements that have nonlinear capabilities. For more details, see the section on element types that can be used with this procedure.

One implication of these assumptions is that pre-buckling displacements have little to no impact on the structural response. In other words, large deformation effects are not considered in the linear stiffness or stress stiffness matrices.

To illustrate this, consider a strut subjected to axial compression (see Figure 1a). If the strut is slender, it may not fail by simple axial compression. Instead, it can become unstable and deflect laterally (as shown in Figure 1b). This lateral deflection marks the onset of buckling.

A linear eigenvalue buckling analysis provides reliable results when pre-buckling deformations are minimal – for example, in the case of a strut under purely axial compression, where lateral displacements before buckling are effectively zero. However, if the strut is also subjected to an initial horizontal load (as shown in Figure 1c), the predicted buckling load becomes less accurate as the horizontal load increases. This is because the linear eigenvalue buckling analysis assumes a linear elastic response and does not account for large deformation effects. In this example, this means the omission of the additional effects that would be induced in the strut as the horizontal force increases.

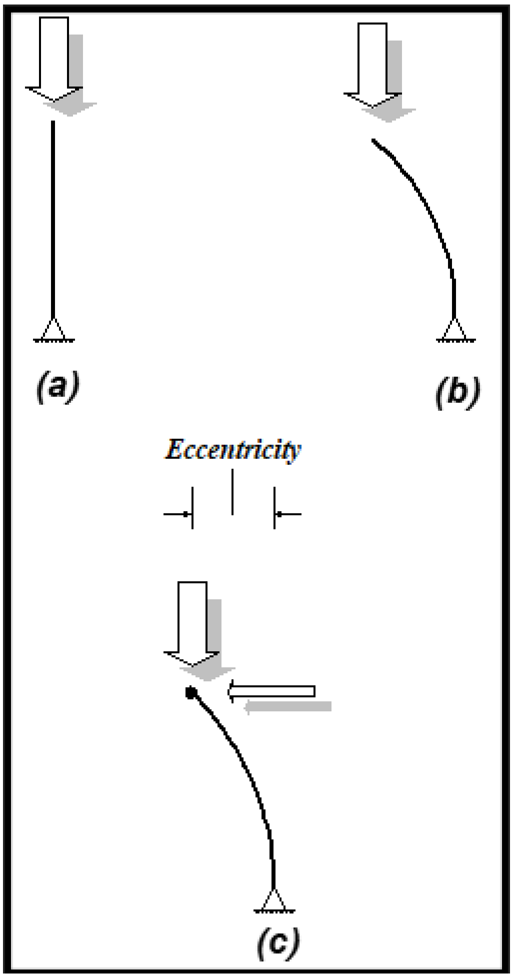

Figure 1 – (a) – (b) Axially compressed strut and (c) axially and laterally loaded strut.

2.2 Application – Structural model

In this application, a typical bridge (Figure 2) consisting of a pair of steel plate girders is considered. Bracing is used to control the buckling lengths of the girders, while stiffeners are included to prevent local buckling. Bearings are idealised as line supports; however, alternative support representations – such as point or surface supports – can also be used. These choices can affect both the structural behaviour and the analysis results, as they may introduce additional flexibility or stiffness depending on the level of support provided.

Thick quadrilateral shell elements with linear interpolation (QTS4) are used to mesh the plate girders, while thick beam elements (BMI21) are used to model the bracing trusses. Both girders are subjected to uniformly distributed loads applied along the centreline of their top flanges.

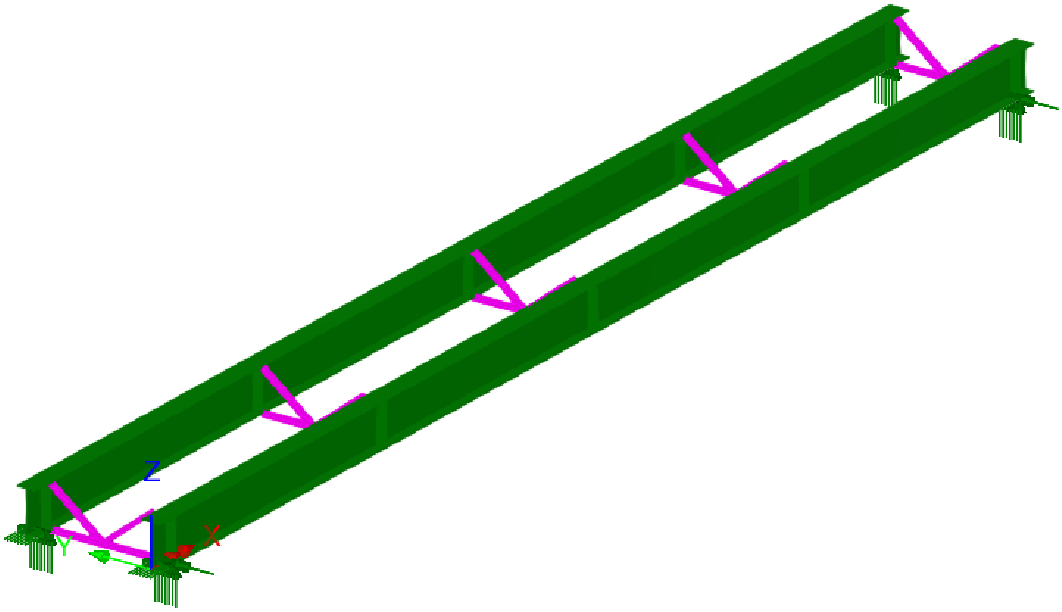

Figure 2 – 3D model of a typical bridge.

2.3 Application – Solver selection

LUSAS provides a range of solvers suitable for different types of analysis. More detailed information can be found in the LUSAS Solver Manual.

In this case, five eigenvalues are calculated using the default eigensolver with standard settings (Figure 3).

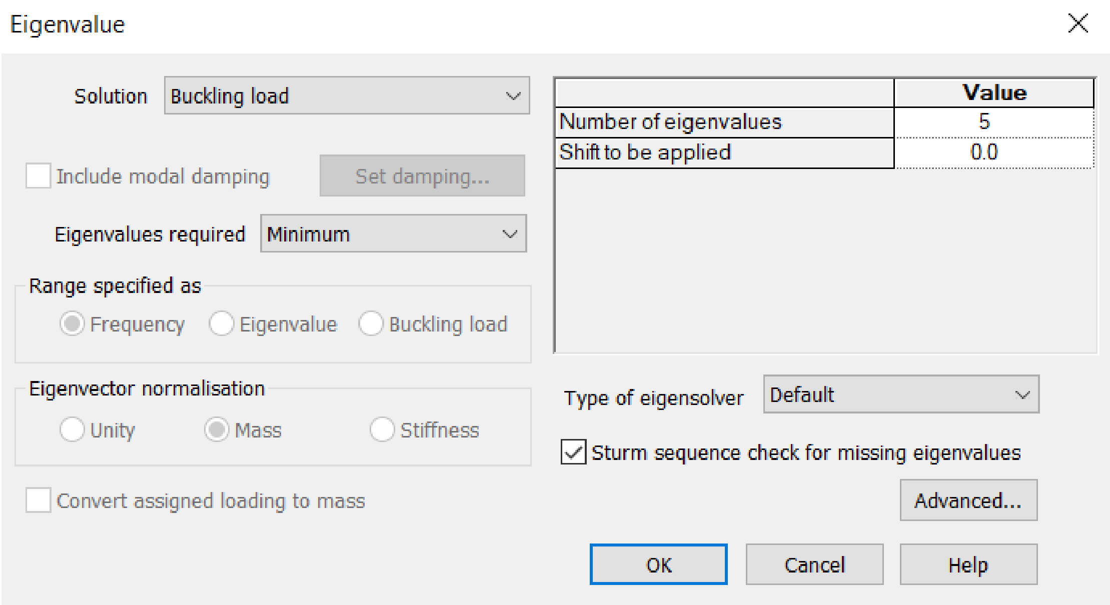

Figure 3 – Eigenvalue controls.

2.4 Application – Results

The results are presented in Figure 4 (*Utilities > Print Results Wizard > Eigenvalues > Loadcases: Active > Eigenvalues (Load Factor)).*

The first eigenmode is shown in Figure 5. A negative eigenvalue was calculated in this case. In such instances, LUSAS generates warning messages in both the Text Window and the output file (refer to Technical Note 1030: Negative Eigenvalues). It is important to ensure that the first positive eigenvalue, typically the most relevant, is computed with a small error norm.

	Mode	Eigenvalue	Load Factor	Error norm
1	1	1.59664	1.59664	0.634706E-6
2	2	-1.80222	-1.80222	6.06693E-9
3	3	1.8205	1.8205	0.204251E-9
4	4	2.14157	2.14157	28.3839E-9
5	5	2.1604	2.1604	0.378673E-6

Figure 4 – Eigenvalue results.

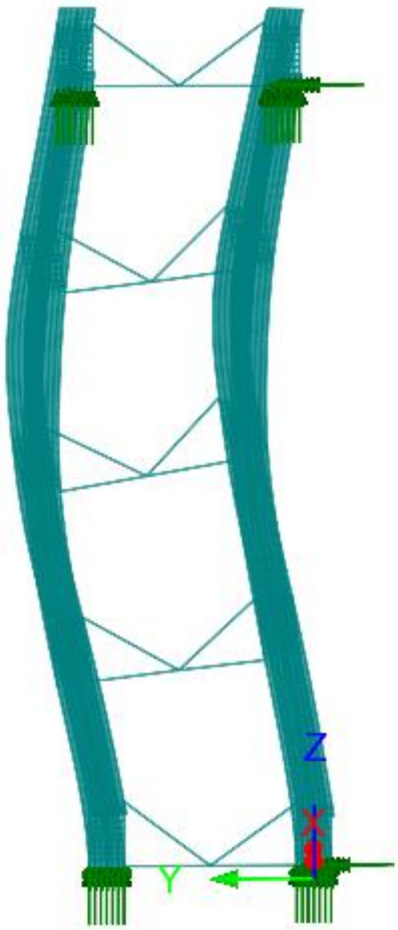

Figure 5 – First buckling mode shape.

Buckling load factors are the factors by which applied loads must be multiplied to cause buckling. In this example, the applied loads need to be multiplied by a factor of 1.5966 to induce buckling (first buckling mode).

Absolute displacement output is not available from eigenvalue analysis. However, displacement results are provided in a normalised form. For buckling analyses, the eigenvectors (mode shapes) are normalised to unity (note that mass normalisation is only supported in eigenvalue frequency analyses). While the mode shapes accurately represent the relative deformation pattern at the onset of buckling, they do not quantitatively define the displacements or stresses of the structure at the buckling load.

Force and reaction outputs are derived from the displacements associated with each eigenvector (mode shape). As a result, these outputs represent the forces that would develop if the eigenvector displacements were applied directly to the structure as prescribed displacement loading, and therefore are generally not useful.

Additionally, the magnitude of the forces and reactions is not quantitative; they are relative to the unit normalised eigenvector and do not represent the forces at the buckling load. To obtain the member forces from an eigenvalue analysis, a further linear elastic analysis must be performed using the same loading conditions, but this time scaled according to the load factor obtained from the buckling analysis.

3. Geometrically Nonlinear Analysis

3.1 General information

A geometrically nonlinear analysis with an appropriate perturbation can account for any pre-buckling displacements and provide a complete structural response. The nonlinear formulation options available in LUSAS include:

- Total Lagrangian
- Updated Lagrangian
- Eulerian
- Co-rotational
- P-Delta

Unlike an eigenvalue buckling analysis, this method does not directly provide the buckling load. Instead, it generates a complete deformation history. The buckling load can then be identified by examining a force–displacement curve at a specific location or across the entire structure.

If no perturbation load is specified, and the applied load alone does not trigger buckling – due to its direction or distribution – then once the structure exceeds the buckling load, a *negative pivot* will appear in the iterative log output. This occurs after the increment has converged (negative pivots that arise during the iterative process at unconverged configurations can be ignored).

Note: If the buckling load for a higher-order mode is exceeded, additional negative pivots may appear - e.g. two negative pivots for the second mode, and so on.

Local and global buckling modes can be accurately captured through a geometrically nonlinear analysis, provided that an appropriate initial perturbation is applied. This approach also directly accounts for the interaction between local and global buckling modes.

3.2 Element types that can be used

In a geometrically nonlinear analysis, only those elements expected to undergo significant nonlinear effects need to support geometric nonlinearity. Linear elements may be used elsewhere in the model. However, if there is any uncertainty, it is recommended to use geometrically nonlinear-capable elements throughout the structure to ensure accurate results.

To verify whether an element supports geometric nonlinearity, refer to the *Element Summary Tables* in the *LUSAS Element Reference Manual*, where you can check whether the required geometric nonlinearity options are ticked for the element of interest.

3.3 Application – Setting up a geometrically nonlinear analysis

A single model in LUSAS can include multiple types of analysis – such as linear elastic, eigenvalue, and nonlinear analyses – which are treated as independent. While a geometric imperfection is not required to perform a geometrically nonlinear analysis, LUSAS allows the use of the deformed shape from one analysis as the initial geometry for another. This feature is particularly useful for simulating the effects of initial imperfections and is the approach used in the application example provided in the following sections.

In the following example, the first eigenmode from the eigenvalue analysis will be used to appropriately perturb the structure. This can be done by opening the Analysis dialog and navigating to the Initial State tab, as shown in Figure 6. In this case, Analysis 2 (geometrically nonlinear analysis) starts with the deformed mesh from Analysis 1 (eigenvalue buckling analysis). The deformed mesh is based on the first buckling mode shape, with a scale factor of 0.01. This scale factor means that the maximum imperfection considered is 0.01 m.

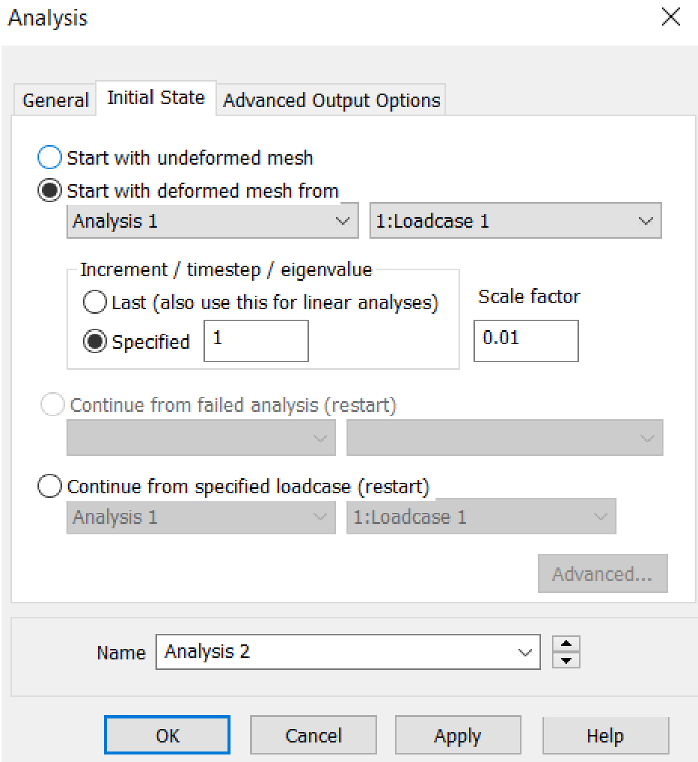

Figure 6 – Initial State tab.

To set up the analysis, the *Nonlinear and Transient* controls should be activated for the newly defined analysis (Analysis 2) and loadcase by right-clicking on the loadcase name. The *Automatic incrementation* option is used to monitor the behaviour of the structure as the loads increase (see Figure 7).

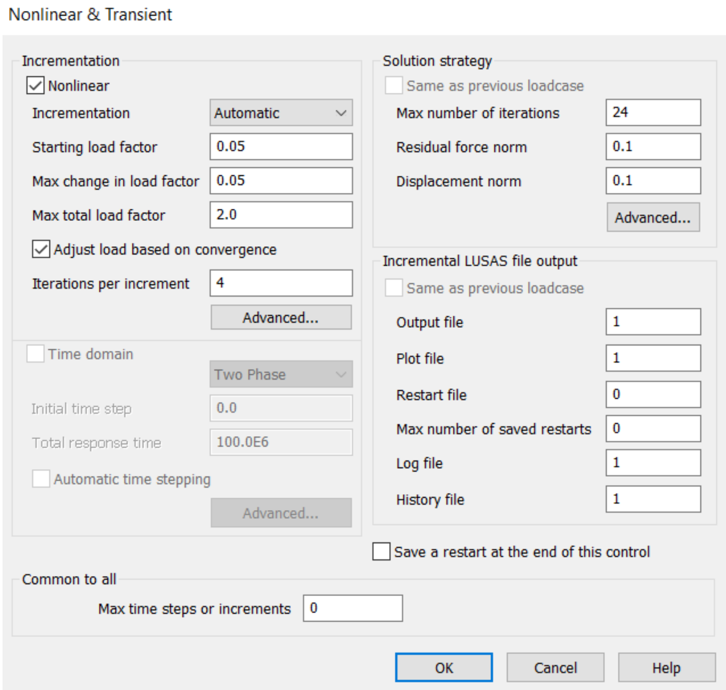

Figure 7 – Nonlinear & Transient dialog.

In the *Nonlinear Analysis Options* dialog (Figure 8), the *Total Lagrangian* and *Co-rotational formulation* options should be selected to enable geometric nonlinearities in the shell and beam elements, respectively. Without these options, nonlinear buckling effects will be ignored.

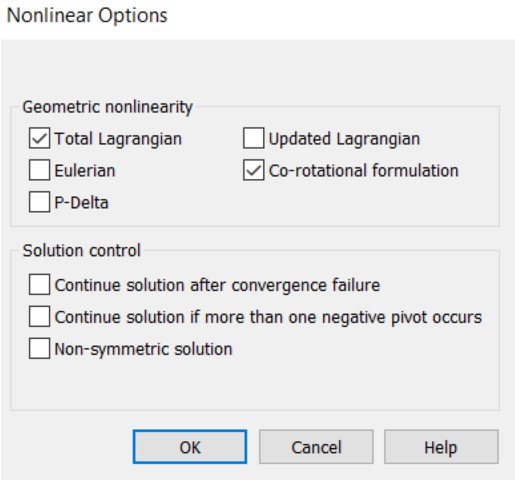

Figure 8 – Nonlinear Analysis Options dialog.

3.4 Application – Results

In a geometrically nonlinear analysis, the buckling load is not provided directly as an output. Instead, it must be inferred from load – displacement curves by identifying nonlinear behaviours – such as a pronounced reduction in stiffness – that may indicate the onset of structural failure. It is also important to note that, in this type of analysis, both displacement and stress magnitudes are physically meaningful, provided the assumptions of the numerical model are valid.

The graph below (Figure 9) shows the lateral displacement of a node located at the midpoint of one of the top flanges, plotted against the total load factor. The maximum load factor reached is 1.48, which is slightly lower than the eigenvalue buckling load factor of 1.60. However, a noticeable reduction in stiffness occurs at a load factor of approximately 1.40. Therefore, an engineer might consider adopting a design buckling load factor of 1.48 or lower. This underscores the role of engineering judgement in interpreting results from a geometrically nonlinear analysis to determine an appropriate buckling load factor.

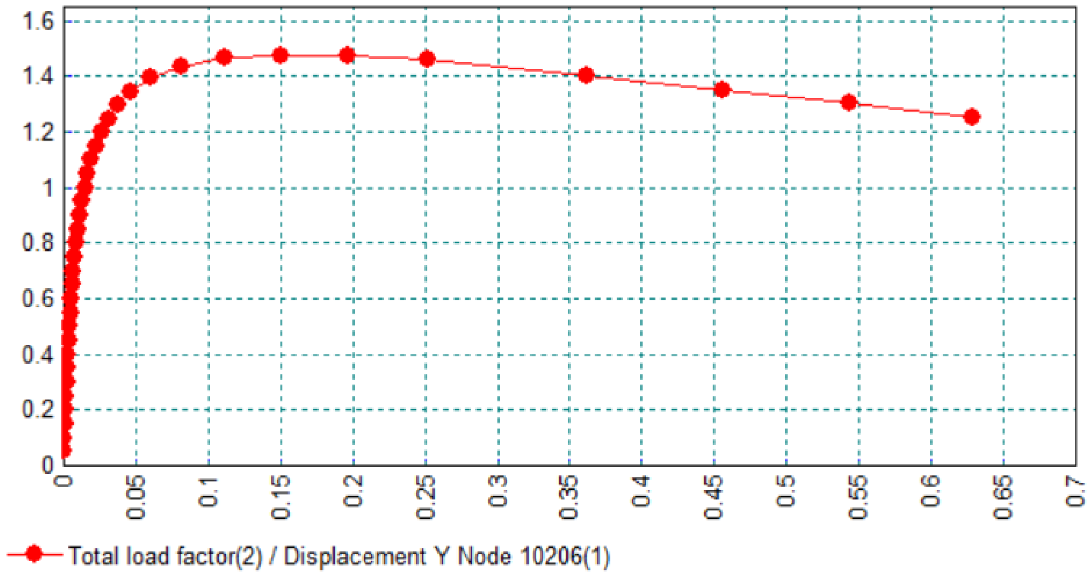

Figure 9 – Total load factor vs lateral displacement of node at midspan graph.

It is also worth noting that the shape of the curve indicates unstable post-buckling behaviour, as evidenced by the descending branch following the buckling point.

4. Geometrically and Materially Nonlinear Analysis

4.1 General information

In a geometrically nonlinear analysis, the material is typically assumed to behave linearly, meaning that stresses can increase indefinitely. However, this is not physically realistic, as real materials have a finite capacity to carry stress. For instance, in steel, stress levels are generally limited by the yield strength, beyond which plastic deformation occurs. To more accurately represent structural behaviour, a fully nonlinear analysis – one that incorporates both geometric and material nonlinearities – is required. This approach better captures the true response of a structure under load.

4.2 Application – Considering material nonlinearity

Material nonlinearity can be incorporated in a third analysis that is identical to Analysis 2, except that the linear elastic material used previously will be replaced with a nonlinear material model. In this example, the nonlinear material attribute will be applied to the shell elements. LUSAS will automatically override the linear elastic material assigned in the *Base Analysis* with the newly defined nonlinear material, ensuring that the updated material behaviour is considered in the third analysis.

The nonlinear steel properties used in Analysis 3 are shown in Figure 10. For ductile materials such as steel, the von Mises yield criterion is generally appropriate. In addition to the initial uniaxial yield stress, a small strain hardening gradient is included to enhance numerical stability and aid convergence. A more pronounced strain hardening can be included, if required, to better capture the material's post-yield behaviour.

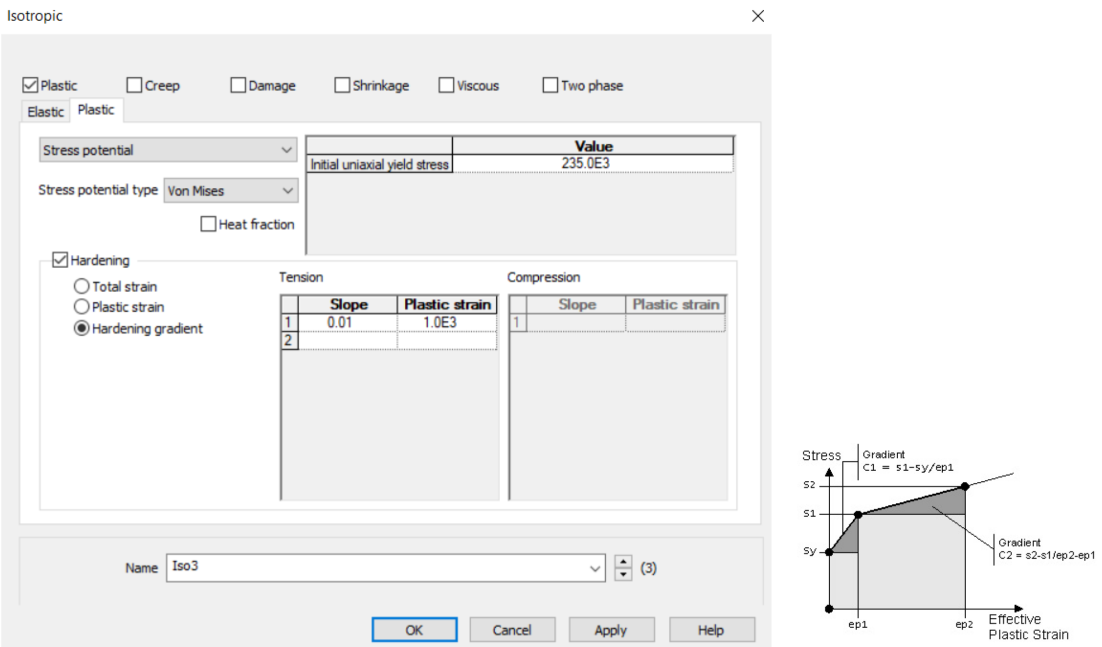

Figure 10 – Nonlinear steel properties.

4.3 Application – Results

Figure 11 displays two curves for comparison: the blue curve corresponds to the results of the geometrically nonlinear analysis (GNIA – Geometrically Nonlinear Analysis with Imperfections), as previously shown in Figure 9, while the red curve represents the results from the geometrically and materially nonlinear analysis (GMNIA – Geometrically and Materially Nonlinear Analysis with Imperfections). By presenting both on the same graph, the impact of material nonlinearity on the structural behaviour in this case becomes clearly apparent.

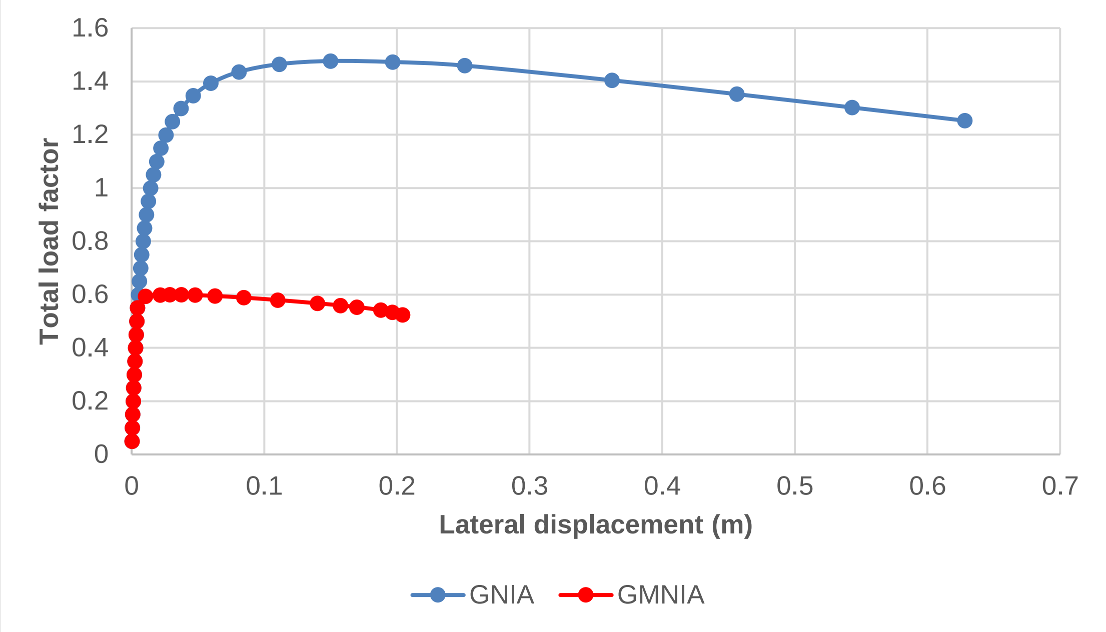

Figure 11 – Total load factor vs lateral displacement of node at midspan graph.

Figure 12 highlights the regions where yielding has occurred, indicated by red asterisks. These asterisks appear at the Gauss points of the shell elements where the von Mises stress has reached the material's yield stress. As shown, significant yielding is observed in both the top and bottom flanges of the two plate girders, particularly concentrated around midspan.

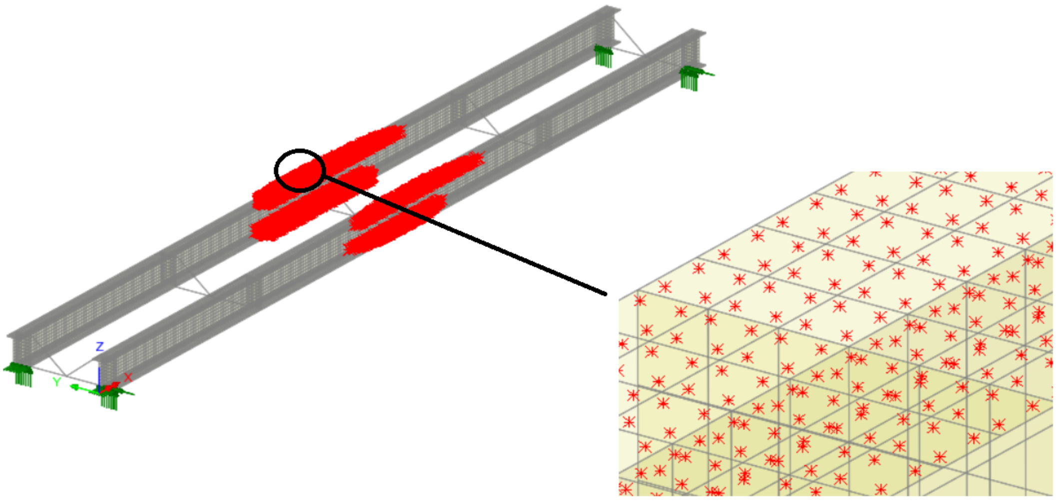

Figure 12 – Areas of yielding marked with red asterisks.

5. Discussion

An eigenvalue buckling analysis provides buckling load factors and their corresponding mode shapes. Buckling load factors are the factors by which applied loads must be multiplied to cause buckling. An eigenvalue buckling analysis typically provides an upper-bound estimate of the buckling load of a structure.

A geometrically nonlinear analysis with an appropriate perturbation can account for any pre-buckling displacements and provide a complete structural response. Unlike an eigenvalue buckling analysis, this method does not directly provide the buckling load. Instead, it generates a complete deformation history. The buckling load can then be identified by examining a force–displacement or stress–strain curve at a specific location or across the entire structure.

In a geometrically nonlinear analysis, the material is typically assumed to behave linearly, meaning that stresses can increase indefinitely. However, this is not physically realistic, as real materials have a finite capacity to carry stress. To more accurately represent structural behaviour, a fully nonlinear analysis – one that incorporates both geometric and material nonlinearities – is required.

A summary of the results discussed in this technical note is provided in Table 1. As noted earlier, a geometrically and materially nonlinear analysis is generally expected to yield results that more closely reflect the actual behaviour of a structure. However, performing separate analyses for each type of nonlinearity can be valuable in identifying which one governs the structural response. In this example, it is evident that material nonlinearity has a more significant influence on the structural behaviour. In other scenarios, however, material nonlinearity may be less significant – for instance, when the material has a high yield strength and the structure is relatively slender, making geometric effects more important.

Method of analysis	Total Load Factor
Eigenvalue buckling	1.60
Geometrically Nonlinear Analysis	1.48
Geometrically and Materially Nonlinear Analysis	0.60

Table 1 Summary of results.

If you have any doubts or require specific advice for your type of analysis, please contact the LUSAS Technical Support team at support@lusas.com.