CUSTOMER SUPPORT NOTE

Buckling Analyses

Note Number:	**CSN/LUSAS/1019**

This support note is issued as a guideline only.

LUSAS

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1. Introduction

Although a linear static analysis will ensure that equilibrium is fully achieved and may also predict stress levels within an acceptable range, the current structural design may still be unsuitable for the intended use. Below the critical buckling load of a structure “stable” equilibrium will usually be achieved, whilst above this load “unstable” equilibrium may result from geometric and/or material effects.

This note concentrates on the methods available in LUSAS to determine the buckling behaviour of a structure; linear, including geometric effects and/or material nonlinearities.

There are three methods in LUSAS to obtain information regarding buckling loads and their respective deformation modes,

- Linear Eigenvalue Buckling Analysis
- Geometrically Nonlinear Analysis
- Full Nonlinear Analysis

The following comments describe the principal characteristics of these methods to assist in the selection of the most appropriate method for the structure to be modelled.

2. Linear Eigenvalue Buckling Analysis

2.1 General

An eigenvalue buckling analysis calculates the linear buckling load factors i.e. the load factors that if applied on the loading the structure will buckle with a specific deformed shape (eigenmode). It should be noted that as eigenvalue buckling analysis is a linear analysis, thus it doesn’t give any information on the post-buckling behaviour (if any) of the structure. I.e. it won’t give any information on whether the structure has a post-buckling resistance, if this is stable or unstable etc, although in certain cases it could calculate with a good approximation the actual buckling load factor of the structure.

This technique can be applied to relatively "stiff" structures to estimate the maximum load that can be supported prior to structural instability or collapse. Determining the overall characteristics of a structure is a matter of engineering judgement but, in general, stiff means stocky (e.g. an engine block). In contrast, an example of a flexible structure would be one manufactured using a significant amount of slender and flexible members. The assumption of a stiff structure, however, is not unreasonable for many applications.

The fundamental assumptions of such an analysis are as follows

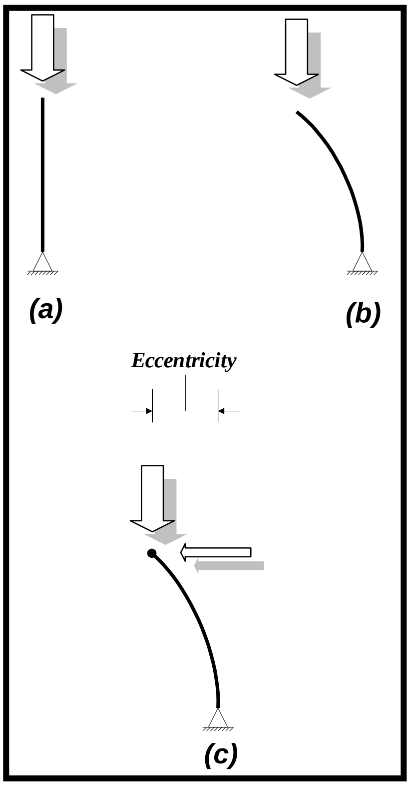
- The linear stiffness matrix does not change prior to buckling
- The stress stiffness matrix is simply a multiple of its initial value

The requirement for a stress stiffness matrix demands a nonlinear software path in LUSAS. Linear eigen value buckling, therefore, is only available for elements that have a nonlinear capability. See the relevant section on the element types available for use with this procedure for further details.

An inference from these assumptions is that the pre-buckling displacements have negligible influence on the structural response. In other words, large deformation effects are not included in either the linear stiffness or the stress stiffness matrices.

To explain this last comment, consider the following strut, loaded axially in compression as shown (a). If the member is slender then, instead of failing by direct compression, it may bend and deflect laterally (b). At this point the member is considered to have buckled.

The linear eigenvalue buckling analysis procedure would yield accurate solutions in this case because the horizontal pre-buckling deformation is negligible (zero).



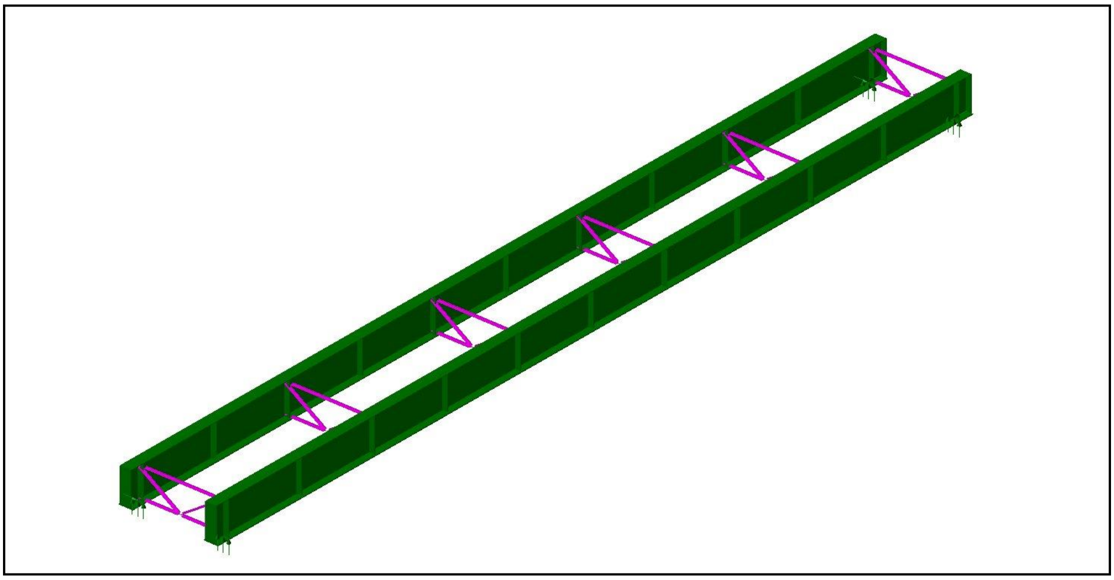
If, however, the strut is also subject to a prior horizontal load (c), the buckling load will be increasingly inaccurate as this load increases. This is because the linear eigenvalue buckling procedure firstly computes the stress state according to a linear elastic procedure. As a result the effects of large deformations are ignored. For the case above, this means the omission of the additional axial forces which would be induced into the strut as the horizontal force increases.

In general, the eigenvalue buckling analysis will, therefore, increasingly overestimate the buckling load as the pre-buckling displacements increase.

This analysis type will provide both local and global buckling modes. However, engineering judgement is necessary to determine which buckling mode is the most critical in order to select the appropriate buckling load factor. It is, of course possible to visually examine the resultant modes in Modeller.

2.2 Structural model

A typical bridge of a pair of plate steel girders is being considered. Bracing is provided to control buckling lengths of the girders and also stiffeners to control local buckling phenomena. Bearings are idealised as line supports, but other approximations e.g. point or surface supports, could be implemented. This would have an impact on the results and behaviour of the structure, as this would be more flexible or stiffer.

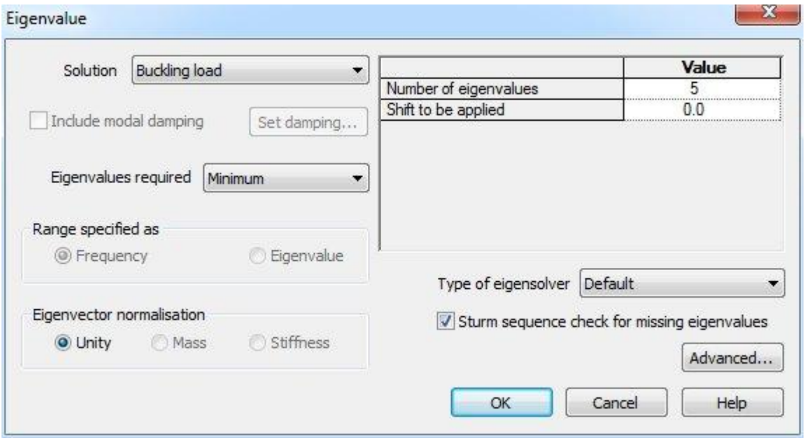


Thick quadrilateral shells with linear interpolation (QTS4) elements are used to mesh the plate girders and thick nonlinear beam (BTS3) elements to mesh the bracing trusses.

2.3 Solver selection

LUSAS includes different solvers that are suitable for each problem. More information on them could be found in LUSAS Solver Manual.

Five eigenvalues are to be calculated using the Default eigensolver in this case, using the default values.



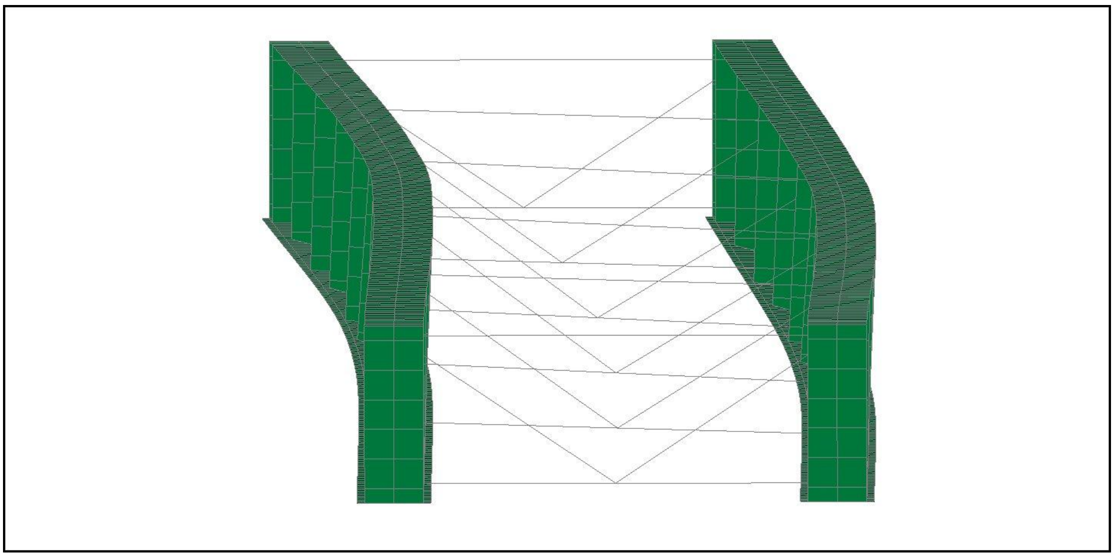
2.4 Results

After solving the model the following eigenvalues are being calculated.

Utilities > Print Results Wizard > Loadcases: Active > Entity: None and Type: Eigenvalues

MODE	EIGENVALUE	LOAD FACTOR	ERROR NORM
1	15.965	15.965	5.09E-10
2	-17.0511	-17.0511	6.61E-9
3	17.1157	17.1157	4.88E-13
4	17.1162	17.1162	4.93E-13
5	17.2041	17.2041	2.75E-10

The first eigenmode is given in the following picture. Negative eigenvalues have been calculated in this case. This is mainly due to numerical difficulties in the solution procedure and LUSAS will return warning messages both in the Text Window and in the output file. Also, it is important that the first positive eigenvalue, which is usually of interest, is being calculated having a small error norm.



By using the alternative eigenvalue buckling solution the negative eigenvalues could be rectified, as this alternative algorithm will always give positive eigenvalues except when the buckling load factor is less than unity.

The buckling load for a mode would be the result of multiplying the actual magnitude of the applied loading (as specified in the LOAD CASE) with the load factor (15.965 in the case of the 1st mode). Note that only one load case (containing any number of different loading types) is permitted in an eigenvalue buckling analysis. As a result, the load factor computed by LUSAS applies to all the loads in this load case. To evaluate the effect of an active buckling load superimposed onto a constant static dead load would require a full geometrically nonlinear analysis to be performed.

Absolute displacement output is not available from any eigenvalue analysis. It is available however in a normalised state. For buckling analyses the eigenvectors (mode shapes) are normalised to unity (mass normalisation is only supported in eigenvalue frequency analyses). The mode shapes are, therefore, accurate representations of the buckling deformation, but do not quantitatively define the displacements or stresses of the structure at the buckling load.

The force and reaction output is evaluated only from a consideration of the displacements associated with each eigenvector (mode shape). The output, therefore, represents the forces developed as if the eigenvector displacements were applied directly to the structure as prescribed displacement loading.

The applied loads are not taken into account at this stage and, moreover, the magnitude of the forces and reactions are not quantitative and are relative to the unit normalised eigenvector - they do not represent forces at the buckling load. To obtain member forces from eigenvalue buckling analyses it is necessary to perform a further linear elastic analysis with the same combination of loading on the structure - but this time factored according to the load factor obtained from the buckling analysis.

3. Buckling using Geometrically Nonlinear Analysis (GNL)

3.1 General

A full geometrically nonlinear (GNL) analysis, if appropriately perturbed, will take account of any pre-buckling displacements of the structure and, moreover, provide a complete response of the structure at all stages of the analysis.

The geometrically nonlinear functionality depends on the element type proposed, but the general available options are as follows:

- *Total Lagrangian*
- *Updated Lagrangian*
- *Eulerian*
- *Co-rotational*

The buckling load is not given directly in this method; rather a complete deformation history is obtained. A graph of force-displacement (stress-strain) at any point or for the structure will enable the buckling load to be determined. See later section on buckling load output for further information.

If a perturbation load is not specified and if the applied load on its own does not induce buckling due to its nature and its direction, then when the buckling load has been exceeded a negative pivot will be found in the iterative log output. This is *after* the increment has converged (negative pivots that occur *during* the iterative procedure, i.e. at unconverged configurations, can be ignored). Note that, if the buckling load for the next (higher) mode is surpassed, two negative pivots will occur and so on. The use of the bracketing facility in LUSAS can help isolate the buckling load and also to determine whether a bifurcation or limit point has been encountered. The bracketing procedure also removes the need to know how the structure should be perturbed in order to initiate buckling in the structure.

Local and global buckling modes are, therefore, accurately predicted with a complete geometrically nonlinear analysis with an appropriate initial perturbation. The interaction between the local and the global buckling modes will be accounted for directly in this manner.

3.2 Which elements are permissible?

Two guidelines are important here:

- *All* the elements used in a linear eigenvalue buckling analysis must have a nonlinear functionality as a result of the requirement for the nonlinear stress stiffness matrix.

  Note that this does not mean that geometrically nonlinear effects are accounted for (they are not). As stated above, it is simply a numerical requirement. The linear eigenvalue buckling procedure assumes geometric *linearity*; ignoring the effects of large deformation.

- For a full geometrically nonlinear analysis, geometrically nonlinear capability is only required by those elements which are deemed to undergo such effects. Linear elements may be used for the remainder of the structure. If there is uncertainty, *all* elements used should support geometric nonlinearity.

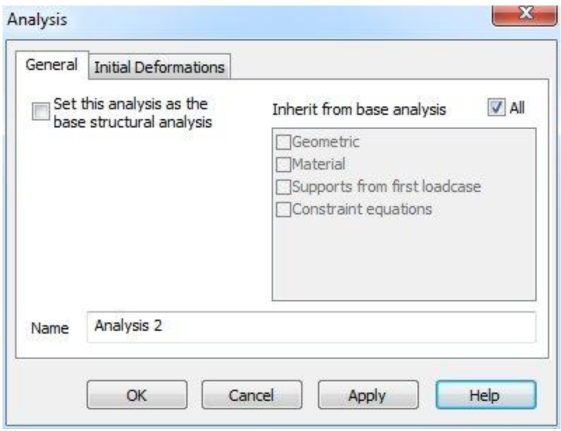
  This procedure depends on geometrically nonlinear functionality to determine buckling effects and cannot be omitted. The LUSAS element reference manual should be inspected to determine which is available for the proposed element(s).

To establish whether a proposed element supports nonlinearity, the Geometric Nonlinearity section in the LUSAS Element Reference Manual should be referred to. Geometric nonlinearity is, as stated above, not used for linear buckling analyses but an affirmative indication in this section is equivalent to saying that nonlinear functionality is available. If this reference states 'Not Applicable' the element does not support any geometric nonlinearity.

3.3 Geometrically nonlinear analysis procedure

Multiple different analyses can take place in LUSAS; linear static, eigenvalue, nonlinear etc. These are independent from each other, but attributes can be inherited from one to another or the deformed shape of an analysis can be used as the initial shape of another.

Analyses > General Structural Analysis > Initial Deformations tab



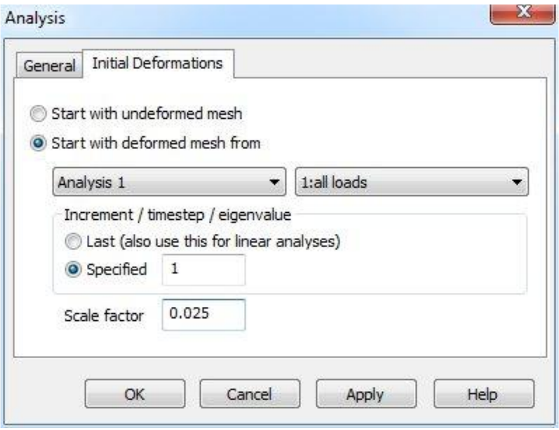
A base analysis is identified as such by a green solve icon alongside its Analysis name in the Analyses Treeview. Other analyses present in the Analyses Treeview that inherit assignments, options and settings of the base analysis are identified by a cyan solve icon.

By default the "*Set the analysis as the base structural analysis*" is not checked (selected). As a result a new analysis will inherit most of the attribute assignments, options and settings etc. of the base analysis.

Deselecting the "*All*" option provides the means to individually select which attribute assignments, options and settings should be inherited from a base analysis and, by their non-selection, which should not.

This procedure won't copy the loads between the analyses, but this could be easily done by copying and pasting them between the loadcases.

In the following example the first eigenmode of the eigenvalue analysis is going to be used to appropriately perturb the structure.

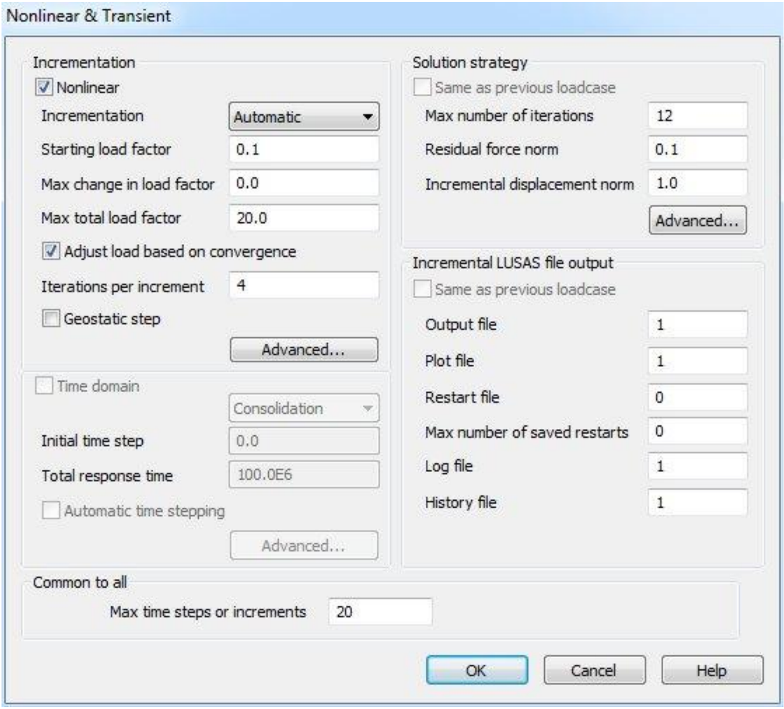


The magnitude of the initial deformations (if it is reasonable) will not affect the total buckling factor calculated from a geometrically nonlinear analysis; only the path followed is going to be changed. This is true whenever a limit point is reached.

The "*Nonlinear and Transient"* controls should be turned on for the newly defined analysis and loadcase by right-clicking on the loadcase name.

Note: An automatic incrementation is specified to monitor the behaviour of the structure while the loads are increasing in the structure. A total load factor greater than the initial eigenvalue buckling load factor is specified in order to monitor the post buckling behaviour of the structure.

In the "*Nonlinear analysis options*" the "*Total Lagrangian*" and "*Co-rotational*" options are selected to enable the geometric nonlinearities in the shells and beams elements, respectively. Without these options, nonlinear buckling will be completely ignored.

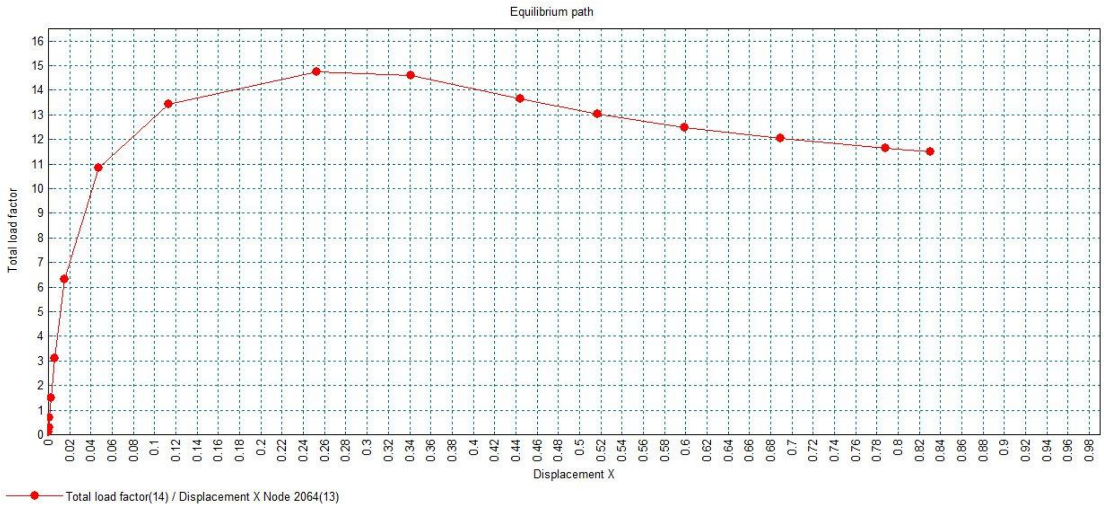


3.4 Results

As the eigenvalue buckling analysis has already been conducted in analysis 1, only the second analysis is selected by default to run, when the "*Solve*" button is pressed.

For a full geometrically nonlinear analysis the buckling load is not automatically output but will require a force-deflection graph for the structural response to be plotted and the buckling load estimated there from; in which case the estimate of P_{cr} could be estimated by inspection. For instance the final graph may be of the form "*nodal lateral displacement vs named total load factor*".

In this type of analysis the magnitude of both the displacements and the stresses are meaningful (according to the assumptions made in the numerical modelling).
The following graph represents the lateral displacements of a node in the middle of one of the two top flanges versus the total load factor.



A decrease in the curve at total load factor 14.47 indicates a change of stiffness and it's the desired value i.e. the buckling load factor. The shape of the curve shows that after buckling the structures has an unstable behaviour i.e. there is a reduction in its stiffness.

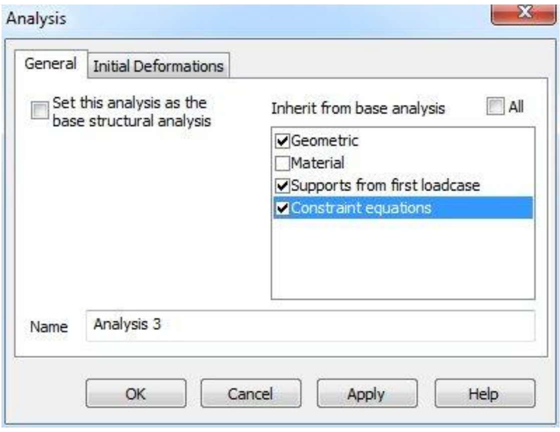
4. Buckling using Material Nonlinear Analysis (MNL)

4.1 General

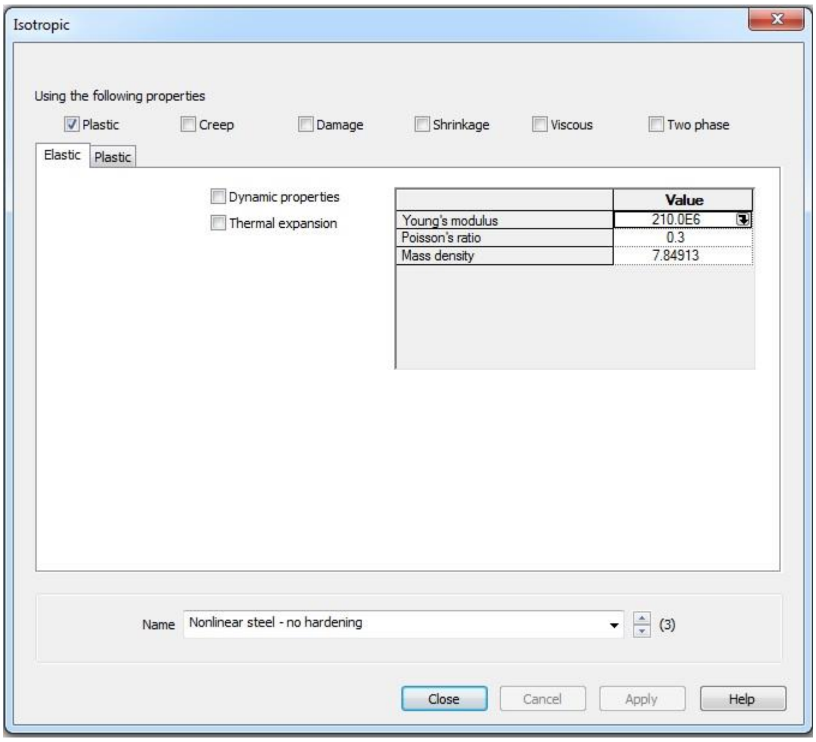
The previous GNL analysis takes into account a linear material; stresses could increase infinitely proportional to displacements without yielding. A full nonlinear analysis would incorporate both geometric and material nonlinearities (as well as boundary nonlinearity if this was significant to the behaviour of the structure).

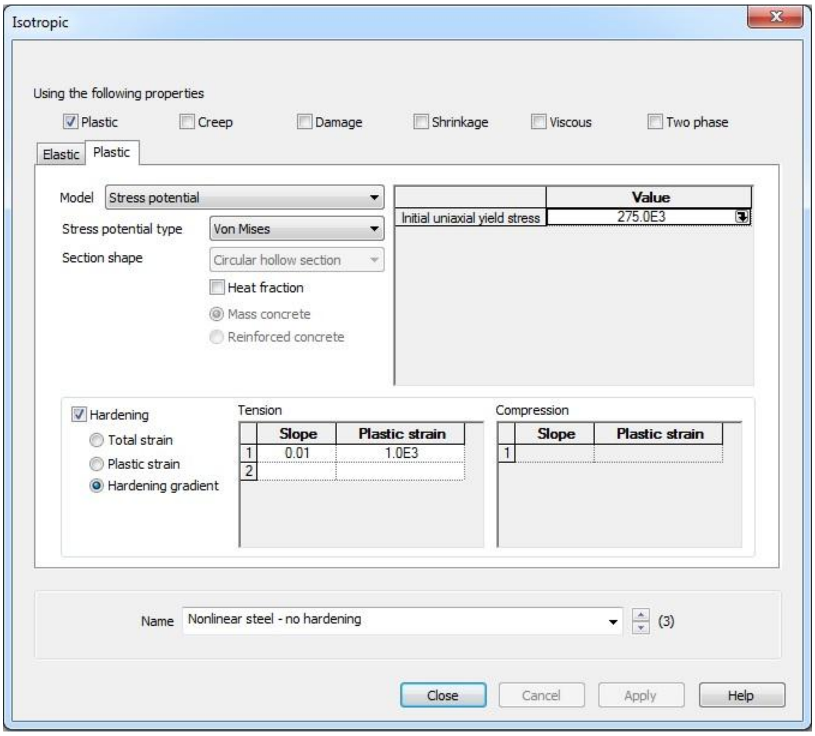
4.2 Material nonlinearity without hardening

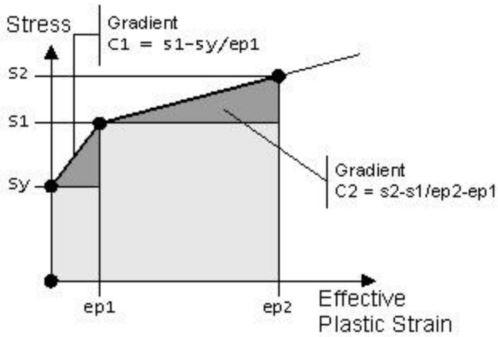
Another analysis can be added, having the same attributes and options as the GNL analysis. Care should be taken that materials should not be inherited from the base analysis as an elastic-perfectly plastic material is to be defined and assigned only onto the shells in the new analysis.



The nonlinear material is defined having a yield stress of f_y = 275MPa and no hardening. The von Mises criterion is used as it models better the behaviour of metals.







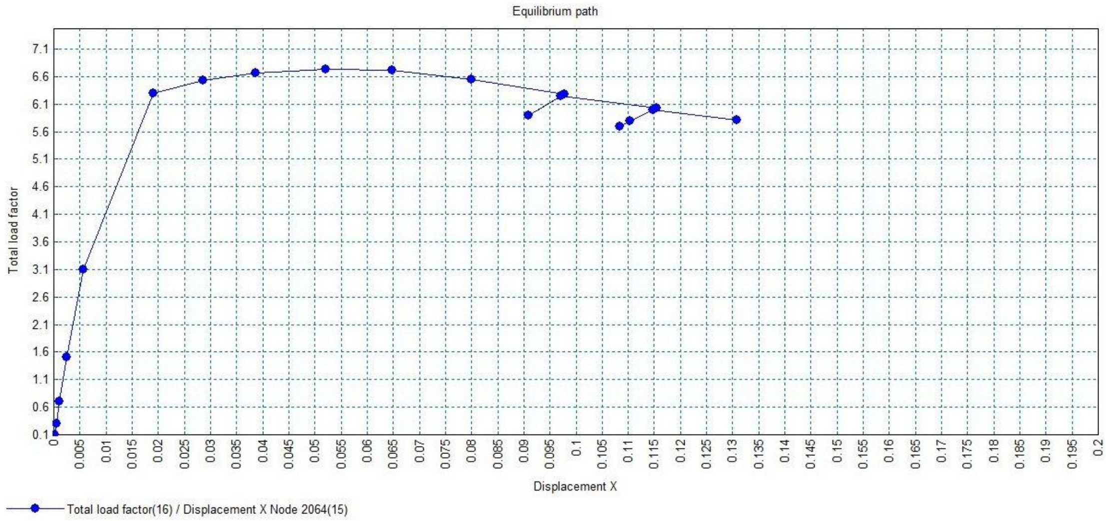
A small hardening slope is being used only for numerical reasons; a “zero” value could potentially lead to numerical instabilities and nonconvergence of the model.

The material attribute is assigned only onto the shell elements in the model and only for the 3rd analysis; beam elements are assigned with a linear material.

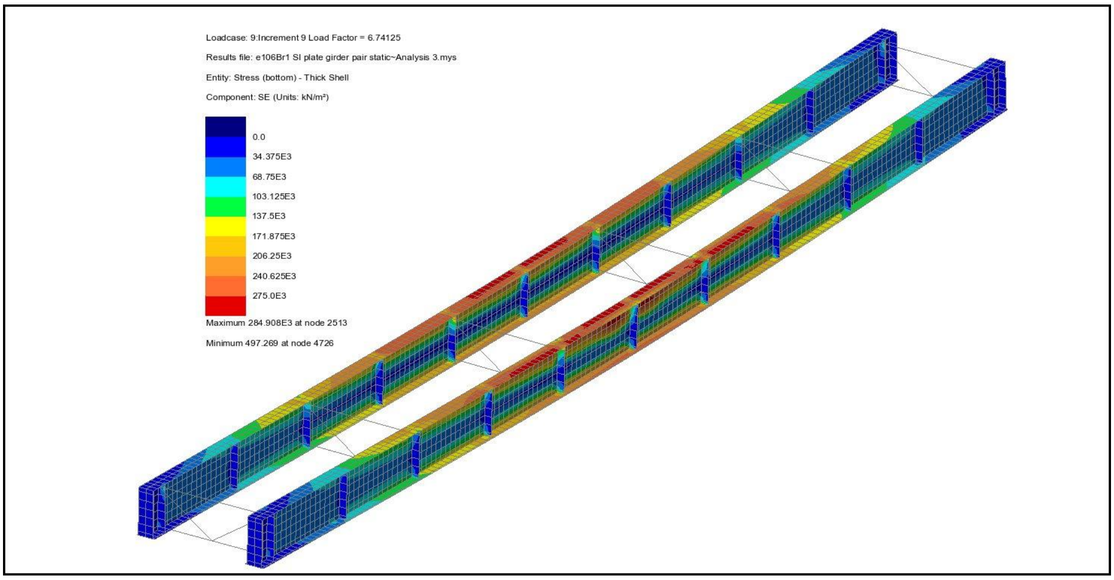
4.3 Results

The same diagram is produced for the same node, as in the previous analysis. In this case buckling occurs for a total load factor of 6.74, thus much less than the previous case, due to the elastoplastic material. The results show that material nonlinearity is significant in this case and should be considered.

The stress contours of the von Mises stresses show that the top flanges yield in the middle of the girders.

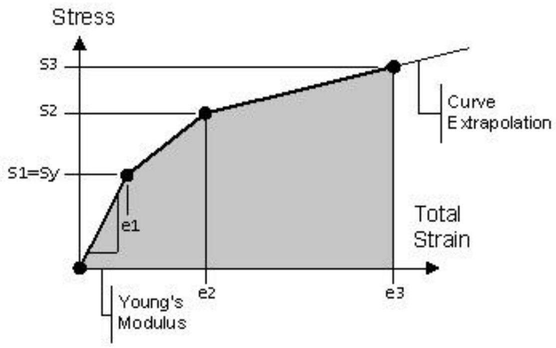


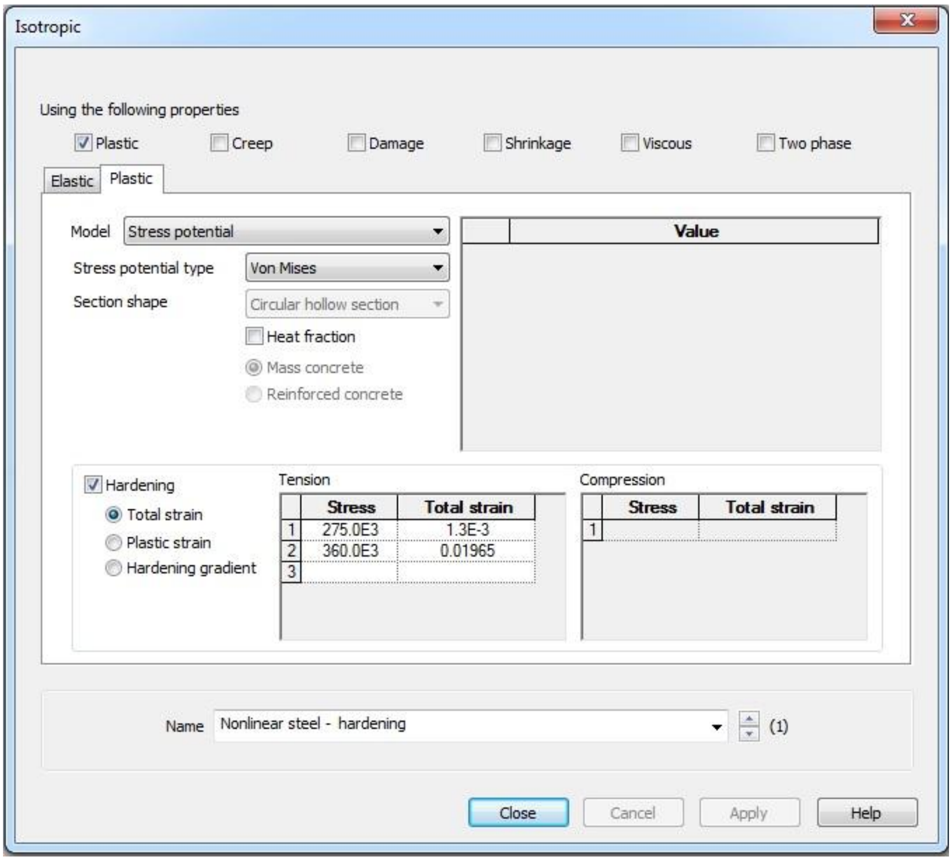
Note that there has been a few small unloading and re-loading increments due to use of Arc-length method for achieving convergence.



4.4 Material nonlinearity with hardening

A fourth analysis is being specified, having the same attributes and options as the GNL analysis and an elastoplastic material with hardening defined and assigned only onto the shells. In this case a different approach for the elastoplastic material is being used for instructive reasons only.

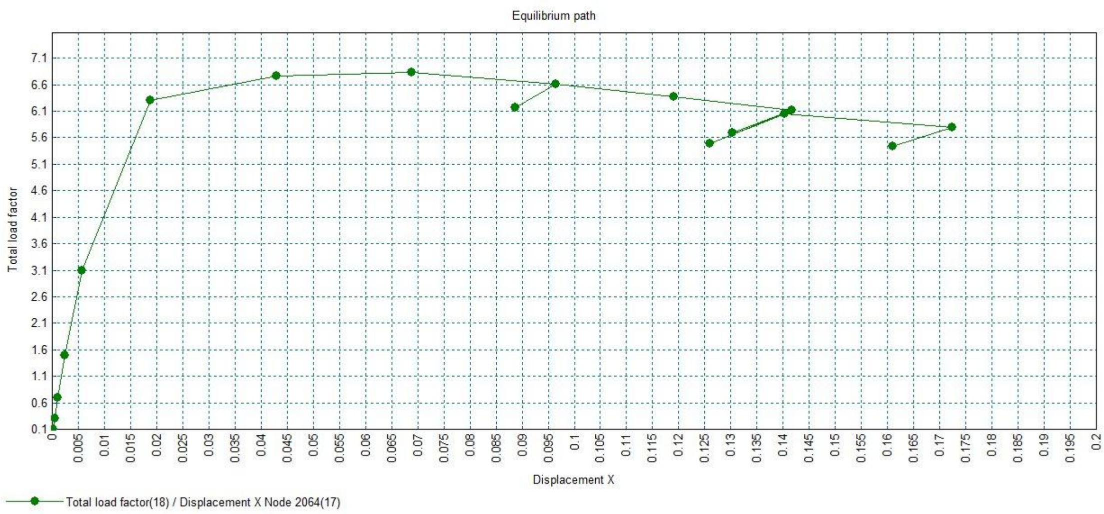




4.5 Results

The "*nodal lateral displacement vs named total load factor*" diagram is produced once again for the same node, as in the previous analysis. Buckling occurs for a total load factor of 6.84, a value slightly larger than the previous case, due to hardening.

The stress contours of the von Mises stresses show that the top flanges yield in the middle of the girders.



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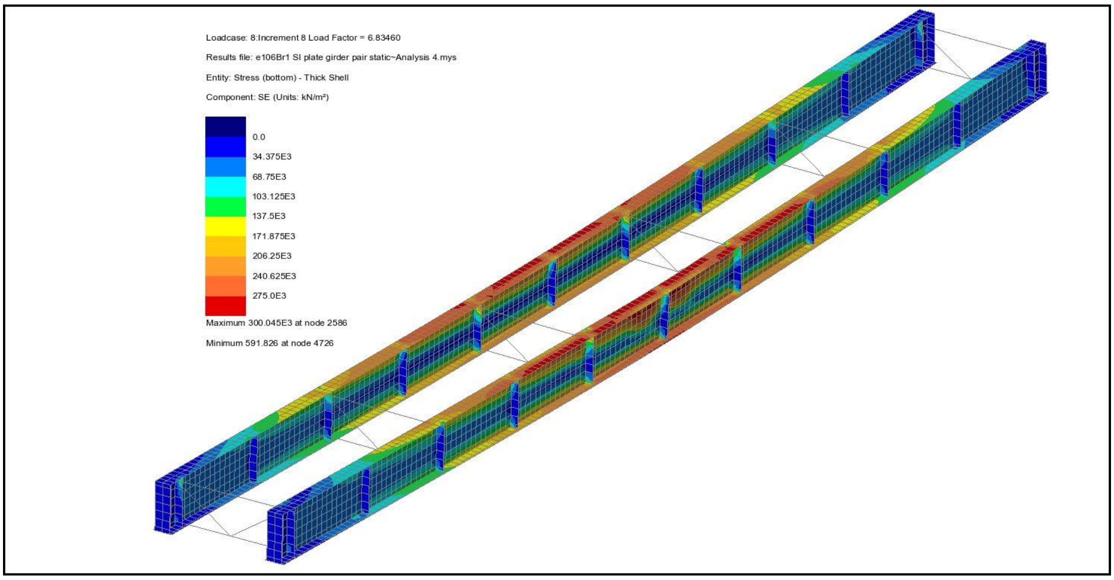


5. Discussion

The eigenvalue buckling analysis of the structure can only be used to provide the mode shape of the structure and the critical buckling load. The stresses and displacements that are obtained are relative to the unit normalised eigenvector and are generally of no practical use. To obtain member forces for the girder it is necessary to perform a further linear static analysis with the same combination of loading on the structure.

The stresses and displacements in the structure when the critical load is applied may be obtained simply by performing a linear static analysis with the loads factored to the buckling load previously derived. These may be compared to other limit state criterion to determine the load carrying capacity of the structure. The critical buckling stress for the mode under consideration will be obtained from the same analysis. If required, this can then be used with reference to design codes to calculate the value of the slenderness parameter for lateral torsional buckling λ_{LT} and the limiting compressive stress σ_{lc}.

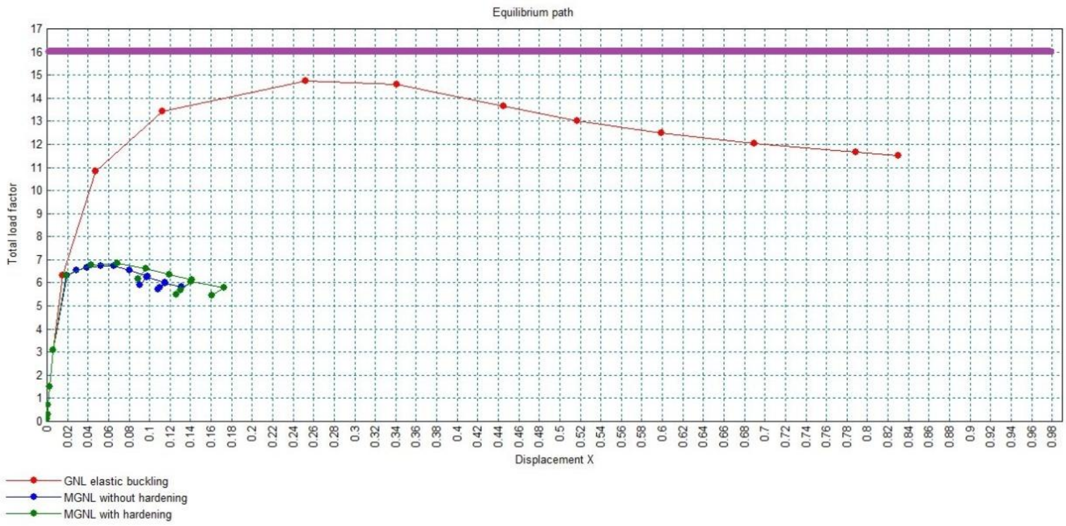
The Eigenvalue analysis provides a bounding factor, which assists in assessment of buckling as a significant design concern; however it is an upper bound ("unsafe") theorem. In the

Eurocode (EN1993-1-1:2005), the Eigenvalue can be used in the determination of design resistance (clause 6.3). [A similar approach also appears in the British Standard, BS5400-3:2000, clause 9.7.5].

The GNL analysis may reduce the total load factor, but it's still considered unsafe, as the material is assumed elastic. It should be noted that the actual value of the imperfection won't affect the total load factor, as long as too large values are not used. Thus, the same value of the total load factor should always be returned and only the initial deformed shape and direction may affect it.

In the simple example which is used in this note, the full nonlinear analysis, including GNL and MNL, reduces more than two times the total load factor. Considering hardening for the elastoplastic material increases slightly the total load factor.

Method of analysis	Total Load Factor
Eigenvalue buckling	15.96
Nonlinear (GNL) elastic buckling	14.47
Nonlinear buckling (MGNL) without hardening	6.74
Nonlinear (MGNL) buckling with hardening	6.84



In the above graph all the curves from the different analyses are plotted together. The total buckling load factor from the eigenvalue analysis is the upper bound of the problem.

6. References

1) EN1993-1-1:2005 "*Eurocode 3: Design of steel structures*"

2) BS5400-3:2000 "*Steel, concrete and composite bridges. Code of practice for design of steel bridges*"

3) EN1993 Practice paper: Buckling analysis of steel bridges – C.Hendy, S.Denton, D.MacKenzie, D.Iles