CUSTOMER SUPPORT NOTE

Using Wood-Armer and Clark-Nielsen Results for Shells

Note Number:	**CSN/LUSAS/1023**

This support note is issued as a guideline only.

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1. Introduction

This support note provides an overview of the application of the Wood-Armer and Clark-Nielsen equations in reinforced concrete slab design (references 1 and 2), and describes how these methods can be employed within LUSAS. Notably, the sandwich model presented in Annex LL of EN 1992-2:2005 is supported in LUSAS and offers improvements over the Wood-Armer and Clark-Nielsen methods. However, this model is not included in the scope of this technical note.

2. Description

2.1 Wood-Armer and Clark-Nielsen equations

In general, a plate element subjected to Mx (bending moment), My (bending moment), and Mxy (twisting moment) is required to be reinforced in the x and y directions. If a slab is designed with reinforcement to resist moments Mx and My only – assuming the twisting moment Mxy is zero – the reinforcement may be adequate along the x- and y- axes. However, when a twisting moment Mxy is present, there is always an orientation in which the principal moments exceed the values of Mx or My. Therefore, a design based solely on Mx and My, without accounting for Mxy, would be inadequate and unsafe.

Wood developed equations that allow the consideration of all three components in slabs with orthogonal reinforcement, while Armer extended the approach to cases with skew reinforcement. Similarly, Clark and Nielsen developed an approach that accounts for in-plane forces (Nx, Ny, and Nxy). The reinforcement required to resist combined bending and in-plane forces is typically determined using a sandwich approach, as proposed by Morley. In this method, the six stress resultants are resolved into two sets of in-plane stress resultants acting on the outer shells of the sandwich.

2.2 Wood-Armer and Clark-Nielsen methods in LUSAS

The Wood-Armer design approach is appropriate for situations with low in-plane (membrane) forces, such as the design of flat, simply supported slabs. In LUSAS, the Wood-Armer moments are denoted as Mx(T), Mx(B), My(T), and My(B), with “T” and “B” referring to the top and bottom surfaces, respectively (relative to the element's local z-axis). For example, the top layer of reinforcement in the x-direction should be designed to resist the moment Mx(T).

By default, the x-direction reinforcement is aligned with each element’s local x-axis. Therefore, to ensure the results obtained are based on the desired orientation, it is often necessary to apply a transformation, such as by using the *Results Transformation* option.

The reinforcement angle input (see Figure 1) specifies the angle (in degrees) between the x-direction reinforcement and the y’-direction reinforcement. Angles are measured counterclockwise from the x-axis towards the y’-axis. Note that the reinforcement at an angle to the x-axis is labelled as y’ to distinguish it from the standard y-axis, which is perpendicular to the x-axis. In My(T) and My(B), the “y” refers to the y’-axis, with the prime symbol omitted for clarity.

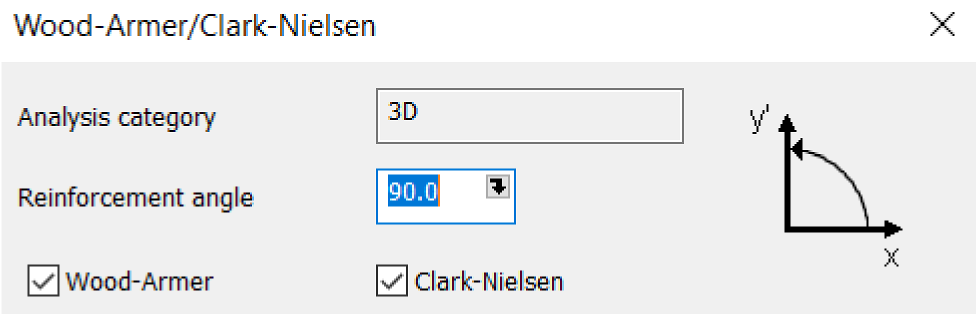

Figure 1 – Reinforcement angle in Wood-Armer/Clark-Nielsen dialog.

The Clark-Nielsen design approach is suitable for structures where in-plane forces are significant. Instead of moments, it provides steel forces (Nx(T), Nx(B), Ny(T), Ny(B)) and principal concrete forces (Fc(T), Fc(B)). In LUSAS, the Clark-Nielsen design approach can also be used when both bending and in-plane forces must be considered, with moments replaced by statically equivalent forces. This is particularly useful in the design of walls and abutments.

3. Illustrative Example

A concrete wall, 10 m long, 5 m high, and 500 mm thick, is subjected to a lateral load of 10 kN/m along its top edge (Figure 2). The base of the wall is fully fixed, preventing both translational and rotational movement. The structure is modelled using QTS4 elements.

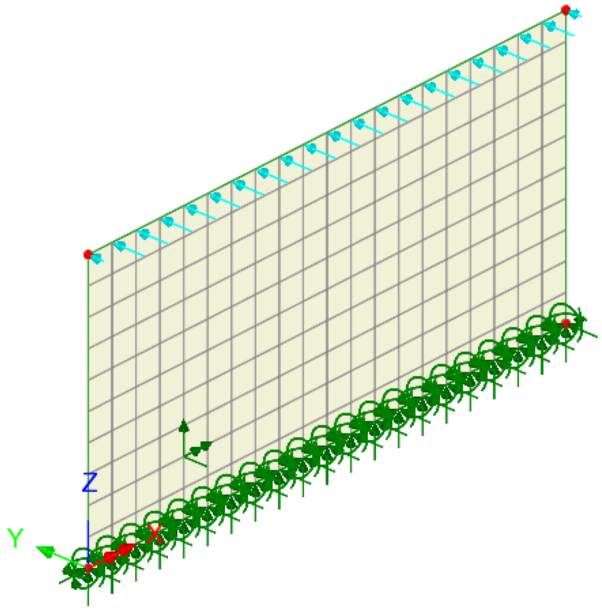

Figure 2 – Concrete wall subjected to lateral load.

The expected bending moment at the base of the structure is approximately 50 kNm/m, closely aligning with the values obtained at the base nodes (Figure 3). Accuracy could be further improved by increasing the number of elements or using QTS8 elements. For this example, the maximum computed value of 48.92 kNm/m is used.

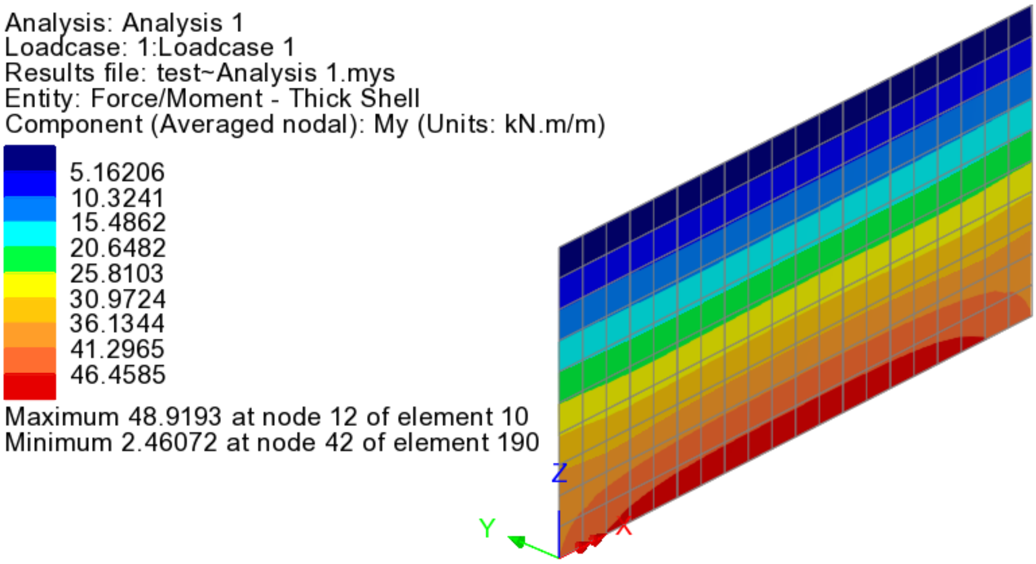

Figure 3 – Bending moment My contour plot.

The *Wood-Armer/Clark-Nielsen* dialog is presented in Figure 4, where orthogonal reinforcement is considered, and the positions of the top and bottom reinforcement bars are specified. A moment of 48.92 kNm/m is expected to induce tensile forces in the reinforcement bars placed in the y-direction of the top face. These forces are calculated as 48.92/(0.5-0.05-0.05)=122.3 kN/m (sandwich analogy). As shown in Figure 5, the Ny(T) value calculated by LUSAS is 122.3 kN/m, confirming this result. The concrete compressive force Fc(B) is equal to -122.3 kN/m.

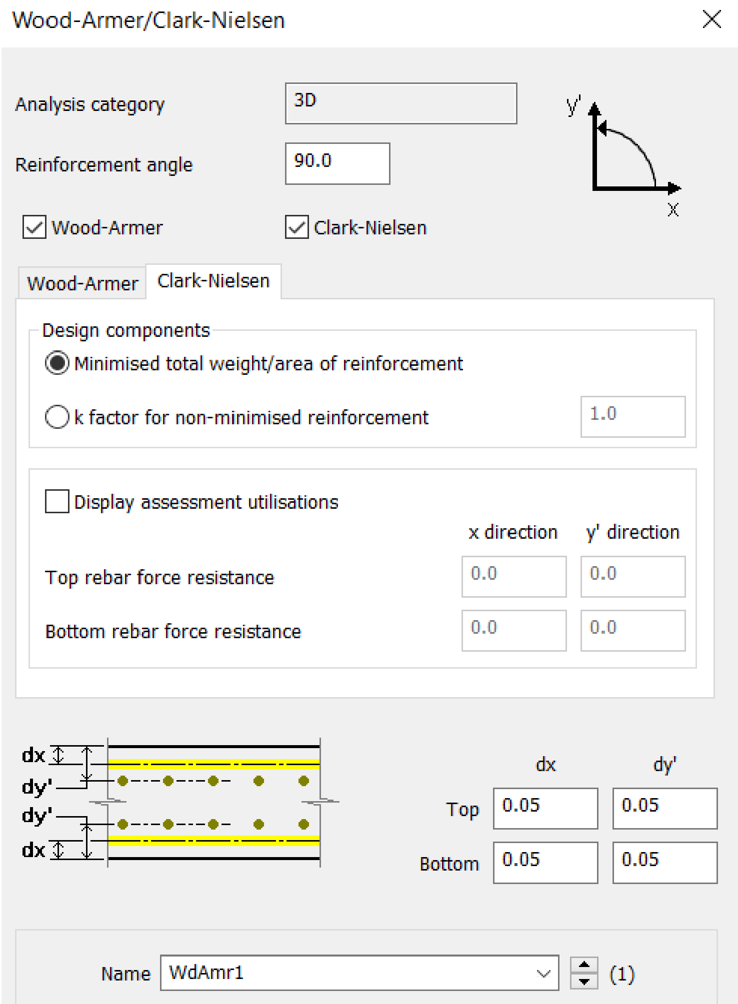

Figure 4 – Wood-Armer/Clark-Nielsen dialog.

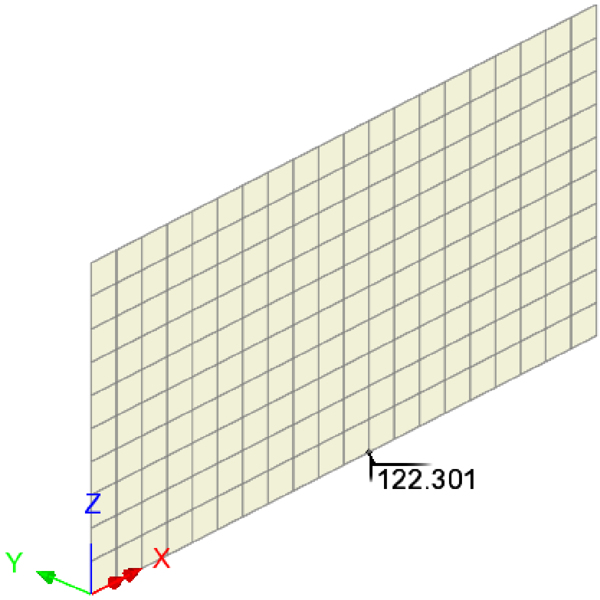

Figure 5 – Ny(T) at selected node.

The introduction of an additional vertical load of 100 kN/m (Figure 6) does not alter the bending effects but generates a compressive membrane force of approximately 100 kN/m. For the sake of this example, a value of -95.11 kN/m is used (Figure 7).

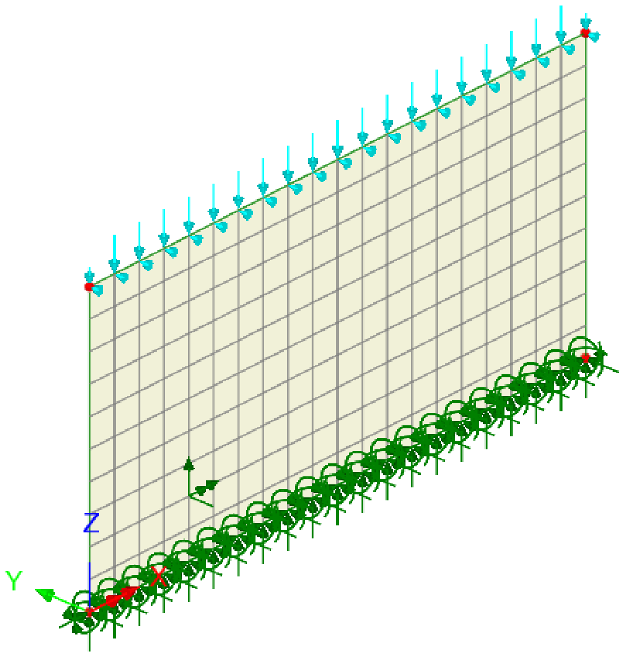

Figure 6 – Concrete wall subjected to vertical and lateral loads.

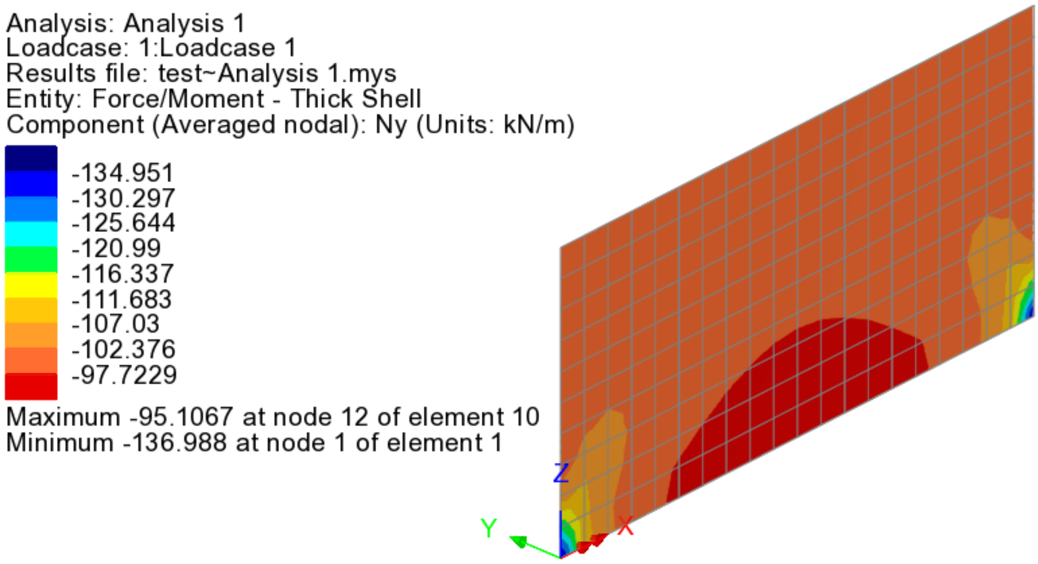

Figure 7 – Membrane force Ny contour plot.

To maintain equilibrium, this additional force is expected to reduce the previously calculated steel tensile force by 95.11/2=47.56 kN/m. Consequently, the force in the reinforcement bars placed in the y-direction of the top face decreases to 74.74 kN/m (Figure 8). This additional force is also expected to affect the previously calculated concrete compressive force, resulting in a value of –169.86 kN/m.

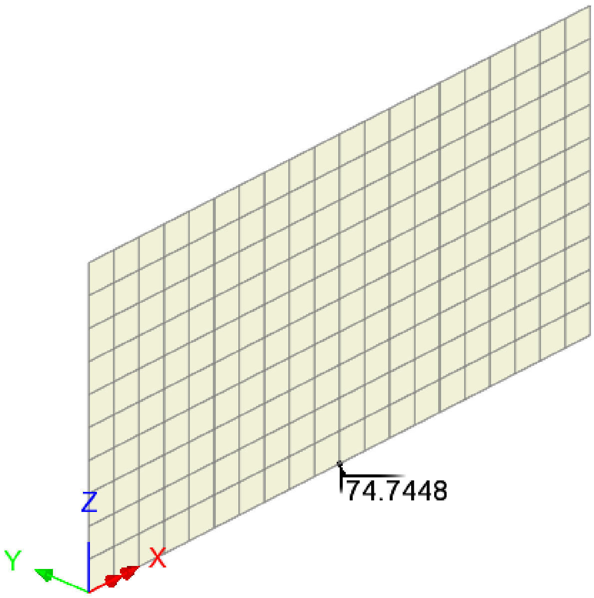

Figure 8 – Ny(T) at selected node.

4. Summary

Wood-Armer and Clark-Nielsen approaches:

1. **Wood-Armer approach:**
   - Appropriate for situations with low in-plane (membrane) forces, such as the design of flat, simply supported slabs.
   - In LUSAS, the Wood-Armer moments are denoted as Mx(T), Mx(B), My(T), and My(B).
2. **Clark-Nielsen approach:**
   - Suitable for structures where in-plane forces are significant.

- In LUSAS, the Clark-Nielsen design approach can also be used when both bending and in-plane forces must be considered, with moments replaced by statically equivalent forces. This is particularly useful in the design of walls and abutments.
- It provides steel forces (Nx(T), Nx(B), Ny(T), Ny(B)) and principal concrete forces (Fc(T), Fc(B)).

It should be noted that Wood-Armer moments are not required for design when Nx(T), Ny(T), Nx(B), and Ny(B) have been calculated using the Morley method followed by the Clark-Nielsen procedure. However, since many design codes are based on the moment capacity of sections, LUSAS also reports Wood-Armer moments, allowing engineers to assess and decide how to address in-plane forces based on the load effects.

Please refer to Section 6.2, "Wood-Armer Reinforcement", in the Theory Manual, Vol 1, for more details. Also, refer to the following pages in our user area for more information:
Index for Wood-Armer & related topics

If you have any doubts or require specific advice for your type of analysis, please contact the LUSAS Technical Support team at support@lusas.com.

5. References

[1] Wood, R.H., "The Reinforcement of Slabs in Accordance with a Pre-Determined Field of Moments", Concrete, V.2, No. 2, 1968, pp. 69-76. (discussion by Armer)

[2] Clark, L.A., "Concrete Bridge Design to BS5400" (Construction Press) Chapter 5 (section entitled "Reinforced Concrete Plates") and Appendix A.