

## CUSTOMER SUPPORT NOTE

# Using Wood-Armer and Clark-Nielsen Results for Shells

Note Number:	<b>CSN/LUSAS/1023</b>
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This support note is issued as a guideline only.



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## 1. Introduction

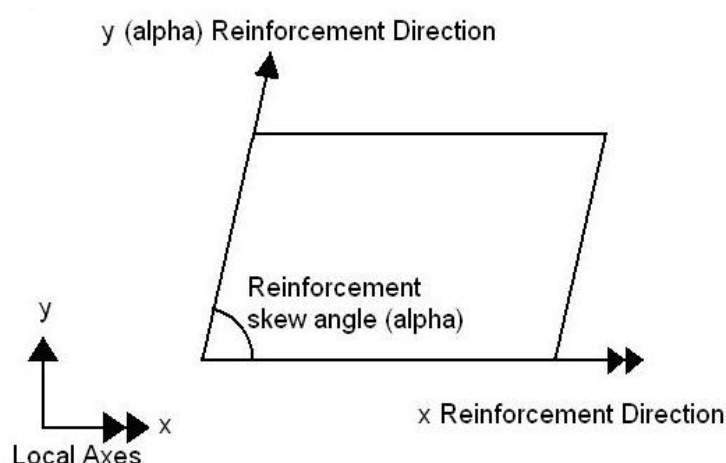
In the design of reinforced concrete slabs, the Wood-Armer moments are a method of accounting for the twisting/torsional moment **M<sub>xy</sub>**. The Wood-Armer method<sup>1</sup> is specifically related to designing slabs where the moment 'triad' (i.e. **M<sub>x</sub>**, **M<sub>y</sub>**, and **M<sub>xy</sub>**) is known. It does not account for in-plane forces.

The alternative Clark-Nielsen method<sup>2</sup> is similar but also includes in-plane forces. It is based on Morley's 'equivalent sandwich analogy' of concrete and is used to design concrete slabs and walls to resist a combination of all six force and moment results (**M<sub>x</sub>**, **M<sub>y</sub>**, **M<sub>xy</sub>**, **N<sub>x</sub>**, **N<sub>y</sub>** and **N<sub>xy</sub>**)

## 2. Description

To summarise the need for Wood-Armer moments, if you designed a slab with reinforcement to sustain moments  $M_x$  and  $M_y$  (i.e. ignoring twisting moment  $M_{xy}$ ) the reinforcement would be adequate in the x and y directions. However, whenever a twisting moment  $M_{xy}$  is present there will always be another orientation (in the principal moment directions) where the  $M_{xy}$  moment is zero and the orthogonal principal moment values are more extreme than  $M_x$  or  $M_y$  (in the actual reinforcement directions). Thus reinforcement designed on the basis of  $M_x$  and  $M_y$  ignoring  $M_{xy}$  would be inadequate and therefore the slab would be unsafe. The Wood method calculates the minimum reinforcement in orthogonal x and y reinforcement directions which results in a slab which is safe for the chosen orientation. Armer later extended the method to be applicable to non-orthogonal reinforcement as well.

In LUSAS the Wood Armer moments are given by  $M_x(T)$ ,  $M_x(B)$ ,  $M_y(T)$  and  $M_y(B)$ , with the 'T's and 'B's representing top and bottom surfaces (please note that top and bottom are defined relative to the element local z-axis). For example, the x-direction top layer of reinforcement should be designed so that the section can resist a moment of  $M_x(T)$ .  $M_y(B)$  and  $M_y(T)$  are given for the transverse reinforcement direction which is in the y (alpha) reinforcement direction at the "Reinforcement skew angle" (alpha) to the components  $M_x(T)$  and  $M_x(B)$  as shown below:



The "Reinforcement skew angle", also known as "alpha", is entered in the "Wood-Armer Options" dialog in LUSAS as shown in section 0 "Illustrative Example". This is the angle between the local x-direction and the skew reinforcement direction measured anti-clockwise from the local x-direction.

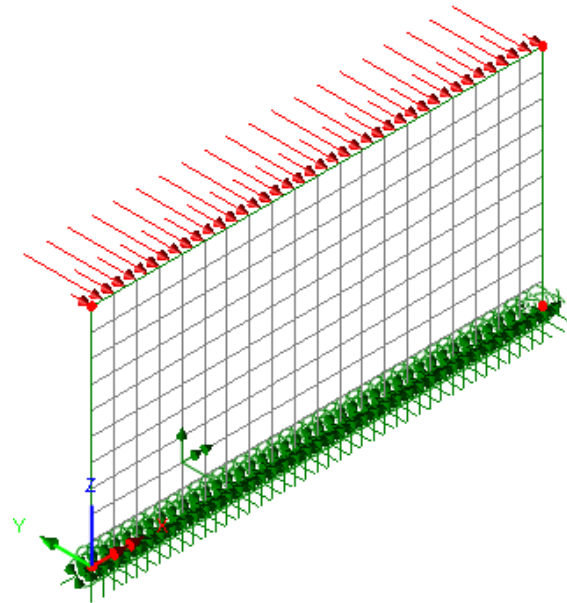
The Wood-Armer design method is appropriate where in-plane (membrane) forces are low – for example flat simply supported slabs. The Clark-Nielsen method extends the theory to also include membrane forces. Rather than moments this method outputs resultant steel forces  $N_x(T)$ ,  $N_x(B)$ ,  $N_y(T)$  and  $N_y(B)$ . In this case the reinforcement should be designed to resist the forces rather than the moments.

Resultant forces  $F_c(T)$  and  $F_c(B)$  are the compressive forces in the concrete per unit width of shell in the direction of maximum compressive stress (on the tension face this will be parallel to the cracks, on the compression face perpendicular to them). In pure bending, this will be equal but opposite to the steel force. If both bending and membrane forces are present the value of  $F_c$  will not be equal to the steel force. They are not required to be directional because unlike the reinforcement the concrete strength is assumed to be equal in all directions.

The Clark-Nielsen method is more generally applicable than the Wood-Armer method because it can be used even if membrane forces are significant. This extends its applicability to walls, abutments etc.

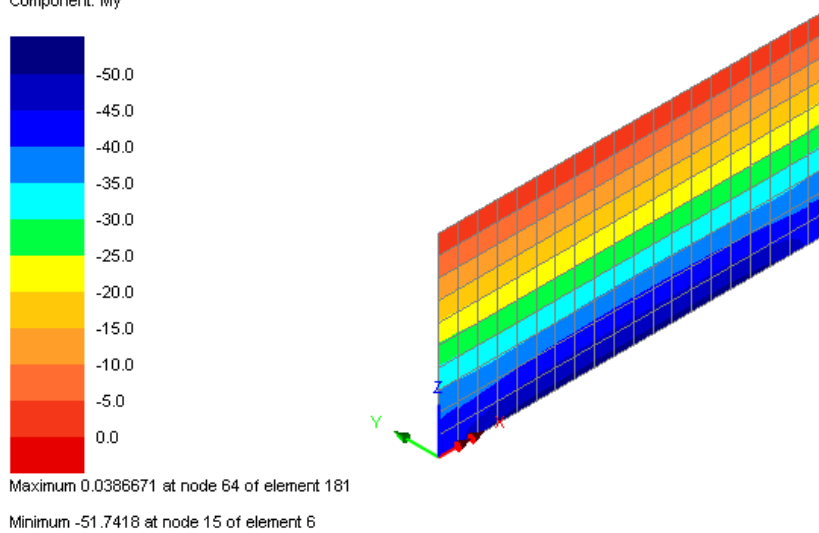
### 3. Illustrative Example

Consider a concrete wall 10m long, 5m high and 500mm thick. The top edge is loaded laterally with a force of 10kN/m.

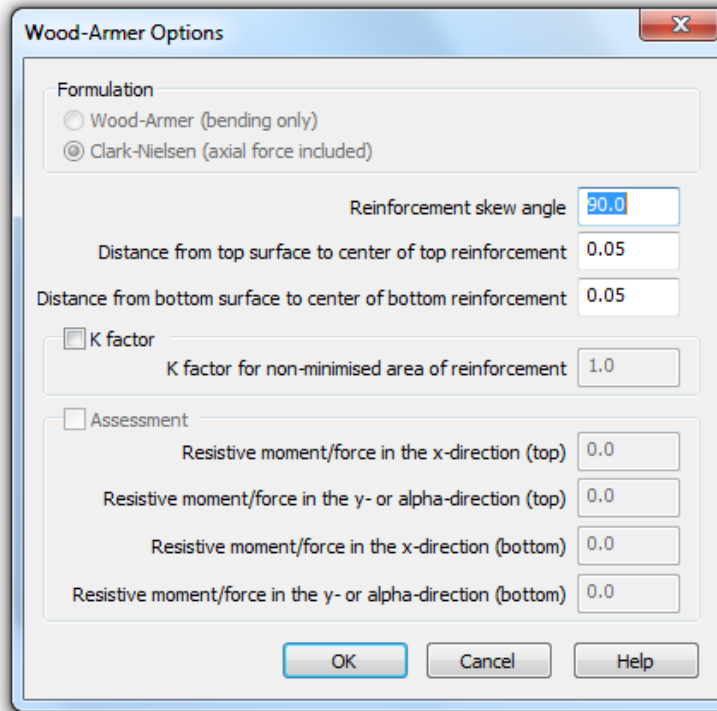


As expected, the y-direction moment **My** at the base is approximately  $5 \times 10 = 50\text{kNm/m}$ :

Loadcase: 1:Loadcase 1  
 Results file: Test.mys  
 Entity: Force/Moment - Thick Shell  
 Component: My



The vertical in-plane force is zero. Due to the direction of the loading and the orientation of the surface axes, tensile forces are expected in the bottom face y-direction reinforcement so Clark-Nielsen component **Ny(B)** is of interest. Orthogonal reinforcement is assumed and distance from top and bottom outer surfaces to centre of reinforcement are set to 50mm by pressing the 'Wood-Armer' button on the contour layer properties dialog.



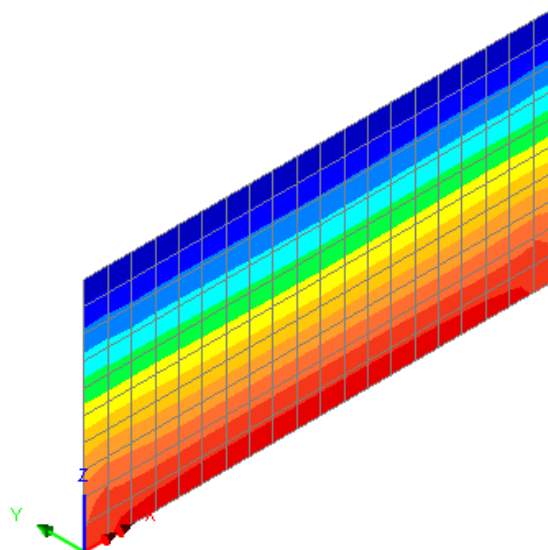
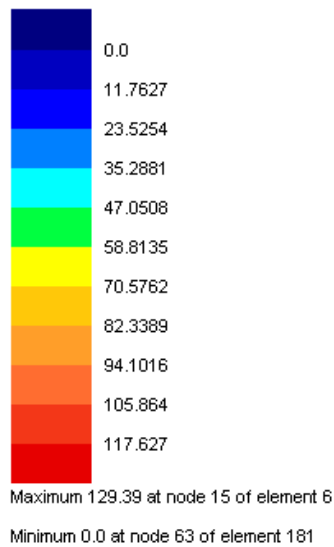
The Morley sandwich analogy assumes a lever arm of the section depth minus the top and bottom cover (i.e.  $500 - 50 - 50 = 400\text{mm} = 0.4\text{m}$  in this case) therefore the expected maximum value of **Ny(B)** corresponding to the maximum moment of  $51.74\text{kNm/m}$  is  $51.74 / 0.4 = 129.35\text{kN/m}$ :

Loadcase: 1:Loadcase 1

Results file: Test.mys

Entity: Force/Moment - Thick Shell

Component: Ny(B)



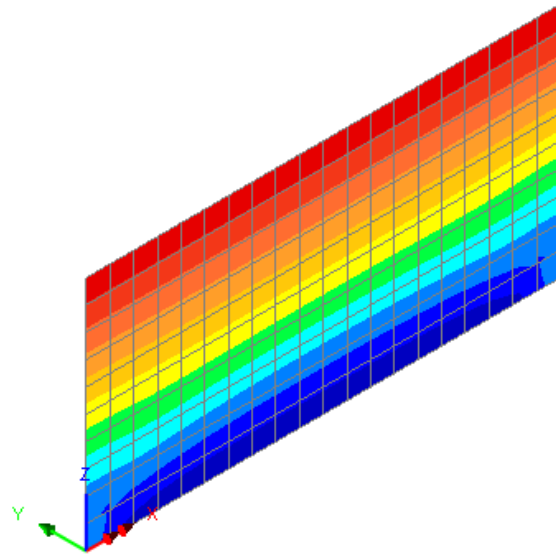
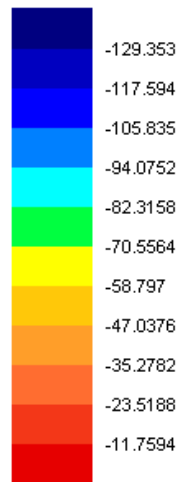
Because there are no in-plane forces, the compressive concrete force at the top surface of the concrete is expected to be equal and opposite to the steel force:

Loadcase: 1:Loadcase 1

Results file: Test.mys

Entity: Force/Moment - Thick Shell

Component: Fc(T)

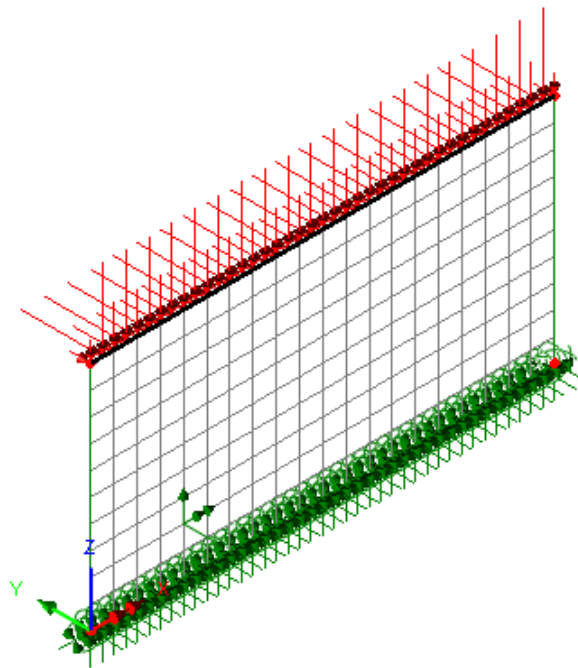


Maximum -1.10709E-3 at node 66 of element 182

Minimum -129.355 at node 15 of element 6

...so both results are as expected.

If a vertical force of 100kN/m is added into the model:



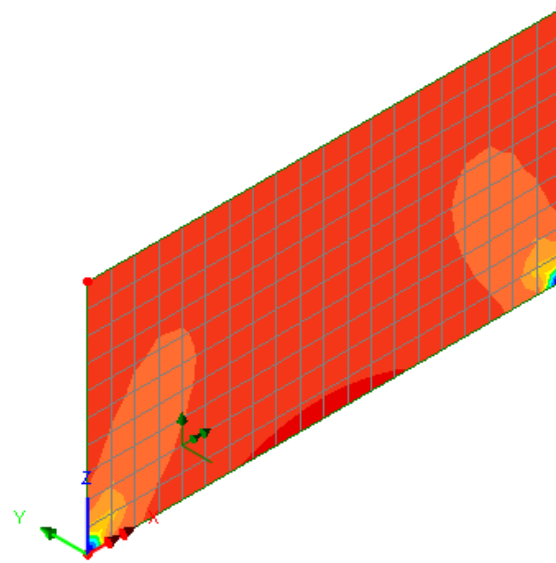
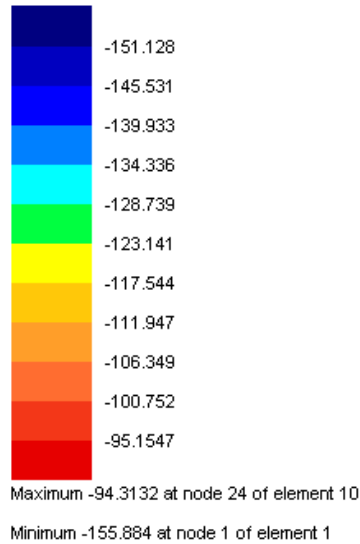
...the bending effects will not be changed but a compressive membrane force of approximately 100kN/m is introduced. The axial force value at the centre base of the wall is 94.3kN/m.

Loadcase: 1:Loadcase 1

Results file: Test.mys

Entity: Force/Moment - Thick Shell

Component: Ny



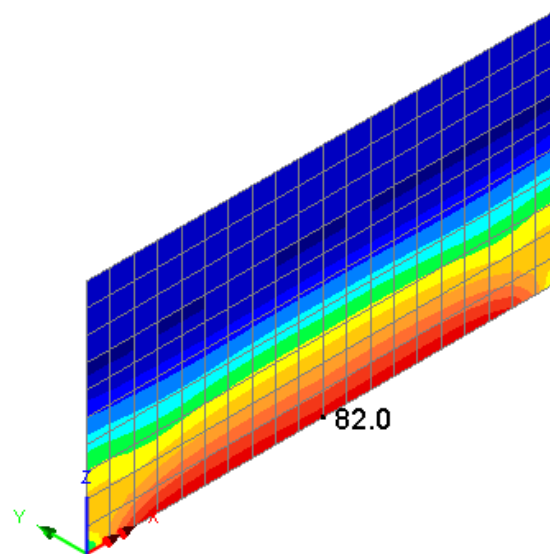
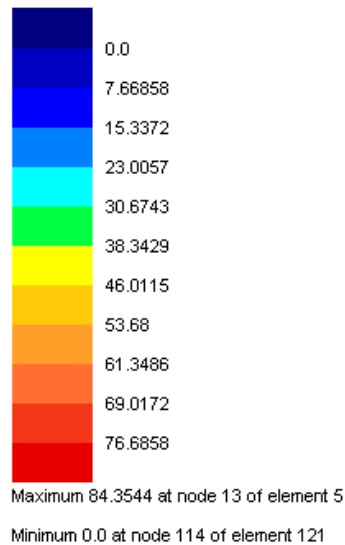
To maintain equilibrium this additional force is expected to result in a decrease in steel tensile force of  $94.3/2 = 47.1\text{kN/m}$  (to  $82.2\text{kN/m}$ ) and an increase in concrete compressive force of  $47.1\text{kN/m}$  (to approximately  $176.4\text{kN/m}$ ).

Loadcase: 1:Loadcase 1

Results file: Test.mys

Entity: Force/Moment - Thick Shell

Component: Ny(B)



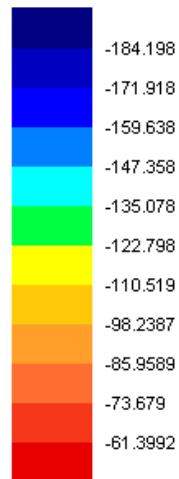


Loadcase: 1:Loadcase 1

Results file: Test.mys

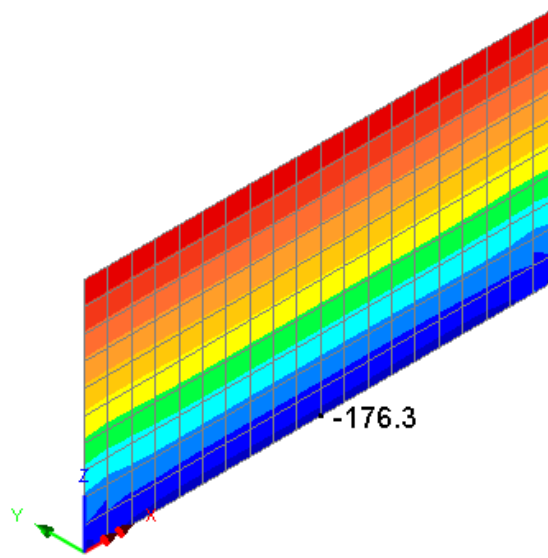
Entity: Force/Moment - Thick Shell

Component: Fc(T)



Maximum -49.9022 at node 64 of element 181

Minimum -184.98 at node 1 of element 1



...therefore the results are as expected.

Please see the section **Theory Manual Vol 1>6.2 Wood-Armer Reinforcement** for details of the calculations of Wood Armer and Clark Nielsen components.

Also see the following pages on our user area for more information:

**Wood-Armer:**

[http://www.lusas.com/protected/faqs/wood\\_armer.html](http://www.lusas.com/protected/faqs/wood_armer.html) - Index of our many Wood-Armer related user area pages.

**Clark-Nielsen:**

[http://www.lusas.com/protected/theory/wood\\_armer\\_clark\\_nielson.html](http://www.lusas.com/protected/theory/wood_armer_clark_nielson.html) - This page gives details of the calculations, an example model and spreadsheet exposing the calculations and references for further reading.

## 4. References

<sup>1</sup> Wood, R.H., "The Reinforcement of Slabs in Accordance with a Pre-Determined Field of Moments," Concrete, V.2, No. 2, 1968, pp. 69-76. (discussion by Armer)

<sup>2</sup> Clark, L.A., "Concrete Bridge Design to BS5400" (Construction Press) Chapter 5 (section entitled "Reinforced Concrete Plates") and Appendix A