CUSTOMER SUPPORT NOTE

Negative Eigenvalues

Note Number:	**CSN/LUSAS/1030**

This support note is issued as a guideline only.

LUSAS

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Table of Contents

1. Introduction

An eigenvalue buckling analysis can produce both positive and negative eigenvalues. This support note provides guidance on how to interpret negative eigenvalues.

2. Description

An eigenvalue buckling analysis provides buckling load factors and their corresponding mode shapes. Buckling load factors are the factors by which applied loads must be multiplied to cause buckling. An eigenvalue buckling analysis assumes linear elastic behaviour and, as a result, typically provides an upper-bound estimate of the structure’s buckling load. However, when a structure is relatively stiff, and geometric and material nonlinear effects are negligible, the computed buckling load may closely approximate the actual load at which buckling occurs. It is also worth mentioning that an eigenvalue buckling analysis does not provide post-buckling information; a nonlinear buckling analysis is required to capture post-buckling behaviour.

An eigenvalue buckling analysis is typically performed before a nonlinear buckling analysis. Buckling mode shapes are often used as initial imperfection patterns in nonlinear buckling analyses.

Before performing an eigenvalue buckling analysis, it is essential to verify the model’s suitability and conditioning. Potential issues arising from modelling errors can be identified by first conducting linear elastic analyses. Key preliminary checks – including evaluating the structure’s deformed shape, deflection magnitudes, and stress distribution – are essential for identifying potential issues before advancing to complex analyses like eigenvalue buckling.

Negative eigenvalues may be calculated as a result of numerical challenges encountered during the solution process. Using an alternative solution method can help eliminate these negative values, ensuring that only positive eigenvalues are calculated (refer to Section 2.8 of the LUSAS Theory Manual, Volume 1). However, it is important to note that negative eigenvalues may have physical importance in some cases, such as reversible load types like wind loading. The following section presents the results of the eigenvalue buckling analysis of a test model, which yields both positive and negative eigenvalues.

3. Illustrative Example

A metallic girder is modelled using QTS8 shell elements (Figure 1). The left end is fixed in translation, while the right end is supported in the Y- and Z-directions. Lateral springs are applied at the locations of the intermediate stiffeners. The structure is subjected to a vertical distributed load.

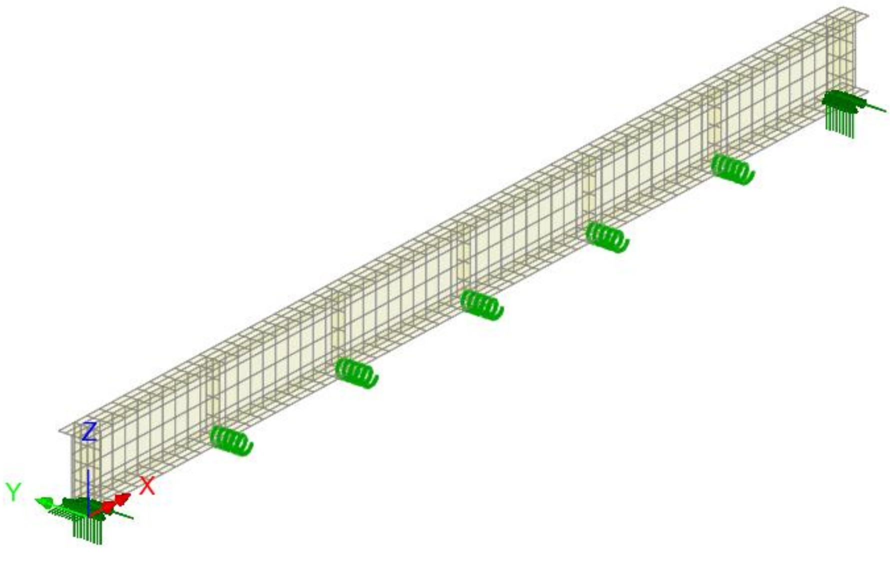

Figure 1 – Metallic girder modelled using shell elements.

An eigenvalue buckling analysis is performed to determine the load factor at which the structure buckles. The eigenvalue settings used in the analysis are shown in Figure 2.

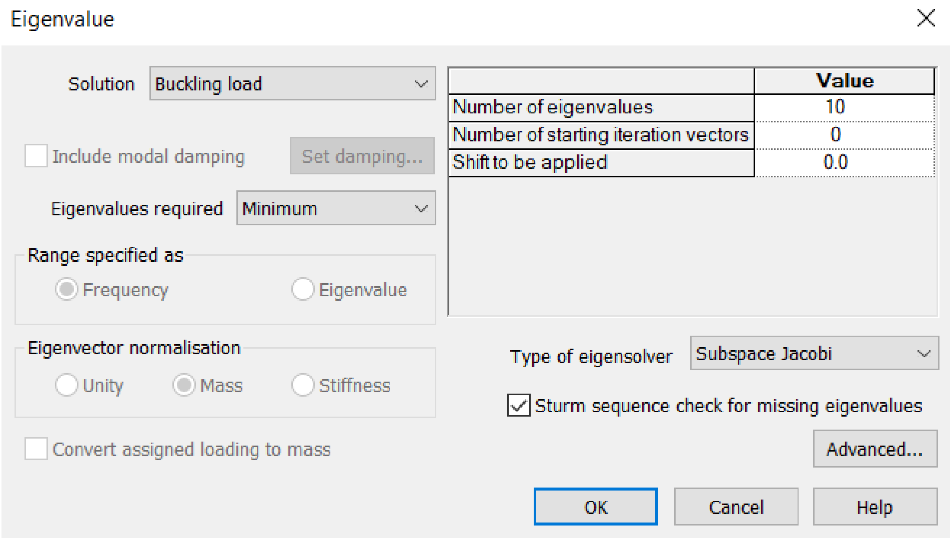

Figure 2 – Eigenvalue settings.

The eigenvalue results can be accessed via *Utilities > Print Results Wizard > Eigenvalues option > Loadcases: Active > Eigenvalues (Load Factor)* and are presented in Figure 3. It is important to note that the error norms for some modes are unusually high, suggesting that the results for these modes may be unreliable. When the error norms exceed the default tolerance, this typically indicates numerical instability. In such cases, LUSAS issues a warning, which is recorded in both the Modeller Text Output window and the output file.

Additionally, the load factors for several modes are negative. It is important to confirm that the first positive load factor (mode 8), which is typically of primary interest, is calculated with a relatively low error norm.

	Mode ▲	Eigenvalue	Load Factor	Error norm
1	1	-17.8779	-17.8779	1.51383
2	2	-13.5035	-13.5035	0.0989278
3	3	-9.23307	-9.23307	5.64788
4	4	-5.9355	-5.9355	3.69812
5	5	-3.06594	-3.06594	0.164106
6	6	-1.37084	-1.37084	0.0679123
7	7	-1.28472	-1.28472	1.35509E-3
8	8	1.42551	1.42551	1.78551E-9
9	9	3.05984	3.05984	0.269528E-9
10	10	5.6891	5.6891	0.246177E-9

Figure 3 – Results presented in table format.

When negative eigenvalues are encountered, it is essential to consider the following key points:

- Reduce the applied load to ensure it is below the lowest expected buckling load.
- Negative eigenvalues may indicate numerical challenges encountered during the solution process or suggest that bifurcation could occur if the load direction is reversed (i.e. the applied load acts in the opposite direction to the load that would cause the structure to buckle).

After reviewing the key points outlined above, if the negative eigenvalues are found to be unrelated to the applied load magnitude or the scenarios discussed in the second key point, they can be disregarded by selecting the *Solve for 1/(1 - buckling load)* option (Figure 4).

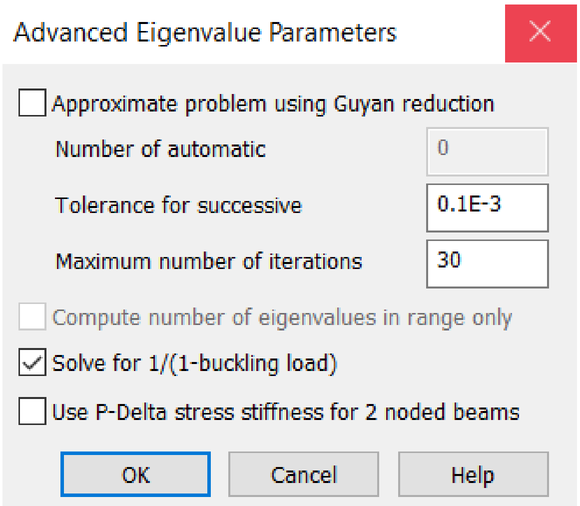

Figure 4 – Advanced Eigenvalue Parameters: Solve for 1/(1-buckling load) option.

4. Solve for 1/(1-buckling load) option

As previously mentioned, the *Solve for 1/(1-buckling load)* option can be used to eliminate negative eigenvalues. However, it's important to keep in mind that negative eigenvalues may still be computed if the applied load is much greater than the buckling load. To avoid this, ensure that the applied load is slightly smaller than the lowest expected buckling load (buckling load factor slightly greater than 1).

In the example from the previous section:

- Select the *Solve for 1/(1-buckling load)* option.
- To ensure that the lowest buckling load factor calculated using the *Solve for 1/(1-buckling load)* option is slightly above 1, multiply the applied load by 1.4.

The results obtained after implementing these changes are shown in Figure 5. The first positive load factor is 1.018, which is slightly greater than 1. The load factor by which the applied load has to be multiplied for the structure to buckle is 1.426 (= 1.018 x 1.4). This value is nearly identical to the first positive load factor presented in Figure 3, verifying its accuracy.

	Mode ▲	Eigenvalue	Load Factor	Error norm
1	1	0.0181087	1.01844	68.2216E-9
2	2	0.542682	2.18666	1.65425E-6
3	3	0.754659	4.07596	2.1265E-3
4	4	0.853959	6.84741	0.132072
5	5	0.962845	26.9143	0.0252455
6	6	0.973403	37.5985	0.0162365
7	7	0.977009	43.4946	0.0127336
8	8	0.981131	52.9967	0.012467
9	9	0.982186	56.1356	0.011824
10	10	0.982536	57.2591	0.0118569

Figure 5 – Results presented in table format – Solve for 1/(1-buckling load) option.

It is also important to check that the error norms computed for the modes of interest are small. As shown in Figure 5, the error norm for mode 1 is indeed small.

5. Range option

Negative eigenvalues can also be eliminated by using the *Range* option to specify positive lower and upper bounds for the eigenvalues to be computed, as illustrated in Figure 6, where the minimum is set to 0 and the maximum to a positive value.

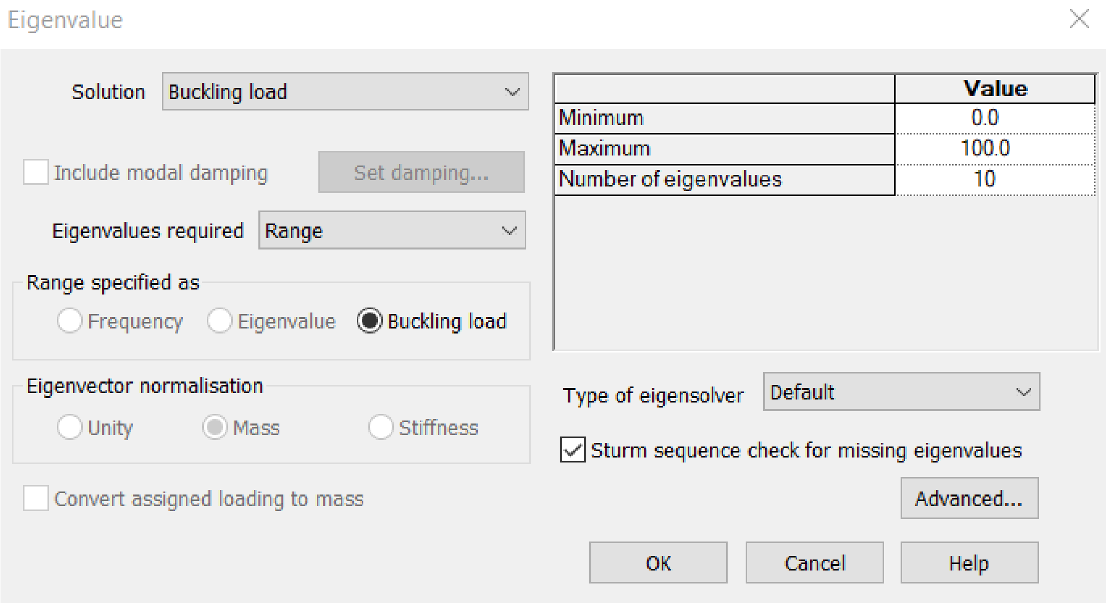

Figure 6 – Eigenvalue settings – Range option.

6. Summary

Negative eigenvalues:

1. **When negative eigenvalues are encountered, it is essential to consider the following key points:**
    - Verify the model's suitability and conditioning. Potential issues arising from modelling errors can be identified by first conducting linear elastic analyses. Key preliminary checks – including evaluating the structure's deformed shape, deflection magnitudes, and stress distribution – are essential for identifying potential issues.
    - Reduce the applied load to ensure it is below the lowest expected buckling load.
    - Negative eigenvalues may indicate numerical challenges encountered during the solution process or suggest that bifurcation could occur if the load direction is reversed.
2. **Solve for 1/(1-buckling load) option:**
    - The *Solve for 1/(1-buckling load)* option can be used to eliminate negative eigenvalues.
    - When using this option:
        - Ensure that the applied load is slightly smaller than the lowest expected buckling load.
        - Check that the error norms computed for the modes of interest are small.

For more details, please refer to the following link:
http://www.lusas.com/protected/instruct/negative_eigenvalues.html

If you have any doubts or require specific advice for your type of analysis, please contact the LUSAS Technical Support team at support@lusas.com.