CUSTOMER SUPPORT NOTE

Negative Eigenvalues

Note Number:	**CSN/LUSAS/1030**

This support note is issued as a guideline only.

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1. Introduction

An eigenvalue buckling analysis calculates the linear buckling load factors. This means that if the applied loads are amplified by the given load factor then the structure buckles with a specific deformed shape (mode shape). An eigenvalue buckling analysis is the first step before commencing to a nonlinear buckling case, if such an analysis is required. Normally the eigenvalue buckling analysis provides an upper bound and in certain cases (when the structure is relatively stiff and geometrically nonlinear effects are not significant) it could closely calculate the actual buckling load factor of the structure.

It should be noted that as eigenvalue buckling analysis is a linear analysis, it does not provide any information on the post-buckling behaviour of the structure.

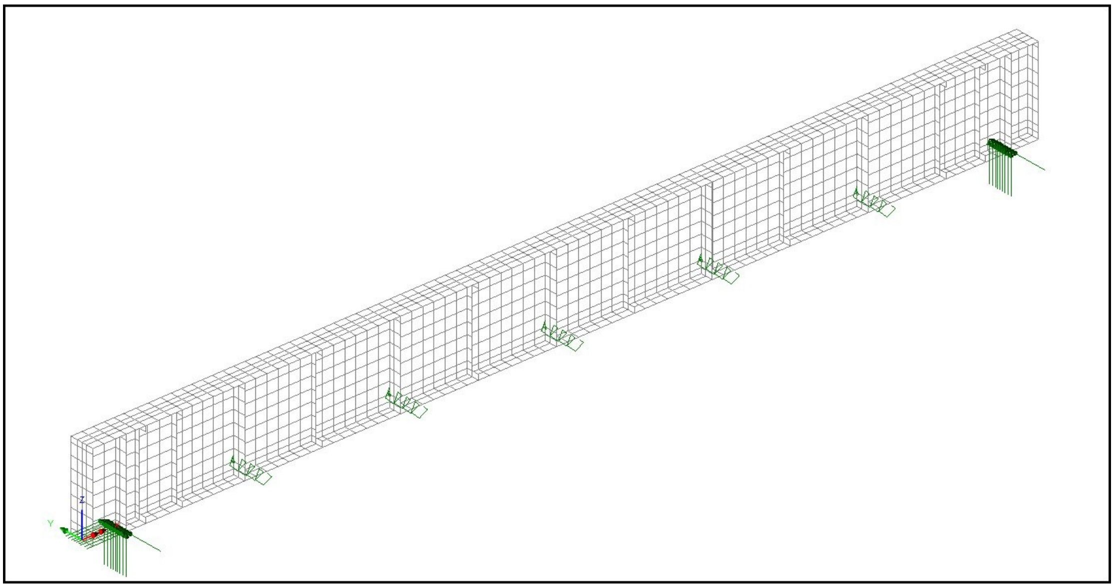
Negative eigenvalues may be computed during an eigenvalue buckling analysis and these can imply genuine numerical difficulties in the solution procedure which can be rectified by using the alternative eigenvalue buckling solution in which the original buckling problem is recast to an alternative form in which, if certain rules are adhered to, all the computed eigenvalues will be positive (see Section 2.8 in the LUSAS **Theory Manual Volume 1** for more information).

This document explains how results that include negative eigenvalues could be interpreted using a test model.

2. Description

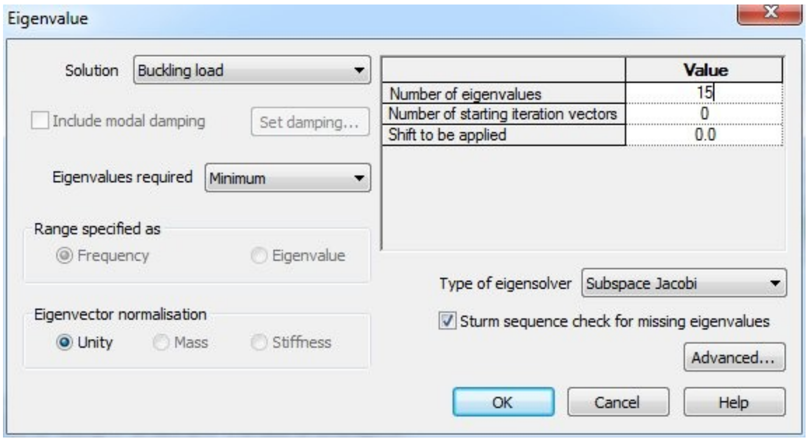
2.1 Sample Structural Model

The following steel I-girder is modelled using quadrilateral thick shell elements with quadratic interpolation (QTS8). Line supports that restrain all translational movements are provided in the left end and line supports that restrain vertical and lateral movements are provided in the right end. Lateral spring supports are also provided at certain parts of the structure.



2.2 Eigen Value Buckling Analysis

Fifteen eigenvalues are to be calculated using the Subspace Jacobi eigensolver using the default values. Note that normally the first (lowest) buckling load factor is in interest.



2.3 Negative Eigenvalues

After solving the model the following eigenvalues are calculated and printed using Print Results Wizard.

Utilities > Print Results Wizard > Loadcases: Active > Entity: None and Type: Eigenvalues

MODE	EIGENVALUE	LOAD FACTOR	ERROR NORM
1	-8.42858	-8.42858	1.25874
2	-6.50724	-6.50724	1.02703
3	-5.04194	-5.04194	0.718942
4	-4.31235	-4.31235	0.794927
5	-4.27981	-4.27981	1.39108
6	-4.08675	-4.08675	2.2855
7	-4.07235	-4.07235	0.647771
8	-3.97362	-3.97362	2.03118
9	-3.83635	-3.83635	1.91206
10	3.18379	3.18379	3.18E-07
11	3.95151	3.95151	1.88E-04
12	4.08383	4.08383	4.01E-04
13	4.23134	4.23134	5.97E-03
14	4.33058	4.33058	6.13E-03
15	4.49847	4.49847	2.80E-02

Negative eigenvalues have been calculated in this case and the error norm for them is above the default tolerance. This is mainly due to numerical difficulties in the numerical solution procedure and LUSAS will return warning messages both in the Modeller Text Window and in the output file.

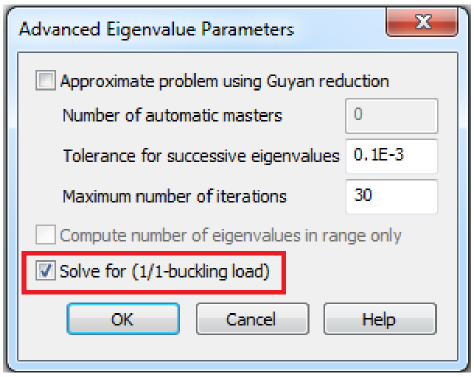
It is important to ensure that that the first positive eigenvalue (mode 10), which is usually of interest, is calculated with a small error norm.

When negative eigenvalues are obtained, at the first instance, the following two key points should be taken under consideration.

- The applied load should be reduced to ensure it is below the lowest expected buckling load factor of the structure
- Negative eigenvalues can indicate bifurcation in tension or bifurcation that would occur if the loading is reversed in sign, i.e. the applied loading is in the opposite direction to that which would cause buckling of the structure

For more specific points you may visit this page:
http://www.lusas.com/protected/instruct/negative_eigenvalues.html

If none of the above is the issue, the **1/(1-buckling load)** option in Eigenvalue Advanced settings can be used to eliminate the negative eigen values.



2.4 The 1/(1-buckling load) Option

Because the load factor for alternative buckling is calculated from 1/(1-eigenvalue), negative eigenvalues can still be computed if the applied load is higher than the buckling load. As a result, the applied load must be modified to ensure that the load factors calculated are close to, but *greater* than unity (i.e. the load applied should be slightly less than the buckling load).

The reason for the recommendation of ensuring that the load factors are *close* to unity is that if the load factor is too large, the eigenvalue will be approaching unity which implies that a small tolerance in the eigenvalue may produce a large error in the computed load factor.

In the specific case of our test, a load factor of **3.15** has been used to factor all the loads acting on the structure.

The procedure to find the load factor that will return a close to but greater than unity buckling factor is a trial-and-error one, meaning that consecutive runs might be required before this is achieved.

The result from this approach is as follow:

MODE	EIGENVALUE	LOAD FACTOR	ERROR NORM
1	1.06E-02	1.01076	2.80E-08
2	0.202908	1.25456	7.28E-08
3	0.228686	1.29649	4.31E-08
4	0.255541	1.34326	4.73E-06
5	0.272612	1.37478	1.42E-05
6	0.299406	1.42736	6.17E-04
7	0.308723	1.4466	8.30E-04
8	0.310831	1.45102	8.14E-04
9	0.399247	1.66458	0.206917
10	0.40749	1.68774	0.113537
11	0.418521	1.71975	6.90E-02
12	0.425367	1.74024	0.211158
13	0.435122	1.77029	8.70E-02
14	0.468423	1.8812	0.25537
15	0.575589	2.35621	0.361342

The first mode shape has a load factor of slightly greater than unity and as the error norm is small the results are considered correct.

To compare with the result in page 2:

1.01076 * 3.15 = 3.18389 this agrees with the previously computed first positive eigenvalue.

It should be noted that the difference in the computed load factors between the **1/(1-buckling load)** approach and the positive eigenvalues without this option is very small.

Generally when negative eigenvalues are calculated and the points mentioned in paragraph 2.3 (and the web page) have been investigated, the negative eigenvalues can be ignored and the first positive eigenvalue could be considered as correct, provided that the error norm is small.