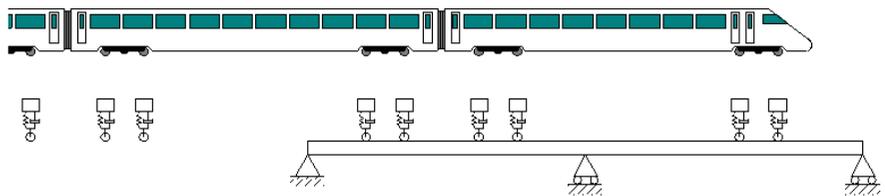


# High Speed Train Modelling through Sprung Masses

For LUSAS version:	19.0
For software product(s):	Any Plus version
With product option(s):	IMDPlus
Note: The example exceeds the limits of the LUSAS Teaching and Training Version and requires approximately 4 GB of disk space for the results files created.	

## Description



This example examines the response of a two-span continuous beam to the passage of a Chinese CRH high speed train modelled as sprung masses based on the paper by Wang *et al*, Latin Journal of Solids and Structures 7 (2010).

Units used are N, m, kg, s, C throughout.

### Objectives

The output requirements of the analysis are:

- Vertical mid-span deflection response of the two spans for a train speed of 250 kph

- Peak mid-span vertical deflections in the Y-direction for train speeds between 25 kph and 600 kph in 25 kph increments
- Vertical mid-span deflection response of the two spans for a train speed of 300 kph, the critical velocity exciting the first resonant frequency of the continuous beam
- Displacement time histories of the body and wheel of the first and last spring-mass systems for a train speed of 300 kph and acceleration time histories of the bodies
- Peak acceleration time histories of all locomotive and passenger bodies over the speed range of 25 kph to 600 kph

### Keywords

2D, Inplane, Moving Mass, Time domain, Response, Interactive Modal Dynamics, IMD, IMDPlus, Eigenvalue.

### Associated Files



- CRH Train.lmd** Attribute library that contains the CRH train definition for the example.
- CRH\_Wheelset\_Positions.xls** Microsoft Excel spreadsheet that contains the wheel positions and spring-mass system identifiers of the CRH train for the example.

## Modelling

### Running LUSAS Modeller

For details of how to run LUSAS Modeller, see the heading *Running LUSAS Modeller* in the Examples Manual Introduction.



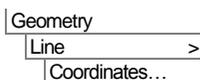
**Note.** This example is written assuming a new LUSAS Modeller session has been started. If continuing from an existing Modeller session select the menu command **File** > **New** to start a new model file. Modeller will prompt for any unsaved data and display the New Model dialog.

### Creating a new model

- Enter the file name as **IMDPlus CRH High Speed Train**
- Choose **User-defined** and press the **Set** button to specify a new folder location. Browse to the **<LUSAS Installation Folder>\Projects** folder, click on the **Make New Folder** button and enter **IMDPlus CRH Analysis** as the folder name and click the **OK** button

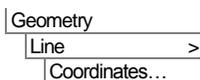
- Ensure an Analysis type of **Structural** is set.
- Select an Analysis Category of **2D Inplane**
- Set Model units of **N,m,kg,s,C**
- Leave the Timescale units as **Seconds**
- Select a Startup template of **None**
- Ensure the Layout grid is set as **None**
- Enter the title as **IMDPlus Sprung Mass Modelling of CRH High Speed Train**
- Click the **OK** button.

## Feature Geometry



 Enter coordinates of **(0, 0)**, **(20,0)** and **(40, 0)** to define the lines representing the main features of the two-span continuous beam.

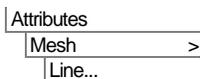
This completes the geometry for the beam that the train will pass across. A further line must also be defined which will be used by IMDPlus to define the path that the train passes along since this line cannot form part of the physical structure.



 Enter coordinates of **(0, 10)** and **(40, 10)** to define the line above the two-span continuous beam that represents the path of the train (the line is defined above the beam line for ease of selection, and the train loading will be projected from this line vertically onto the structure).

## Meshing

The Line features representing the two-span continuous beam are to be meshed using two-dimensional beam elements.



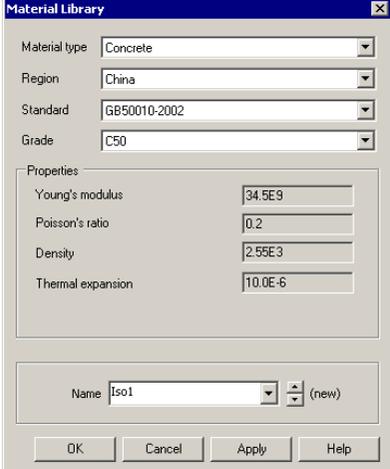
- Select **Thick beam, Linear** elements.
- Set the number of divisions to be **20**
- Enter the attribute name as **Thick Beam** then click **OK**
- Select the lines defining the two-span continuous beam.
- Drag and drop the mesh attribute **Thick Beam** from the  Treeview onto the selected model Lines.

40 BMI2 elements will be created along the lines.

### Material Properties

The Line features representing the two-span continuous beam are to be assigned C50 concrete in accordance with the China National Standard: Code for Design of Concrete Structures.

- Select material type **Concrete**
- Select region **China**
- Select standard **GB50010-2002**
- Select grade **C50**
- Leave the name to be **Iso1** and click **OK** to define the material.
- With the two lines defining the two-span beam still selected, drag and drop the material attribute **Iso1 (C50 | Concrete | China GB50010-2002)** from the  Treeview onto the selected model Lines.



The Material Library dialog box is shown with the following settings:

Property	Value
Material type	Concrete
Region	China
Standard	GB50010-2002
Grade	C50
Properties	
Young's modulus	34.5E9
Poisson's ratio	0.2
Density	2.55E3
Thermal expansion	10.0E-6
Name	Iso1 (new)

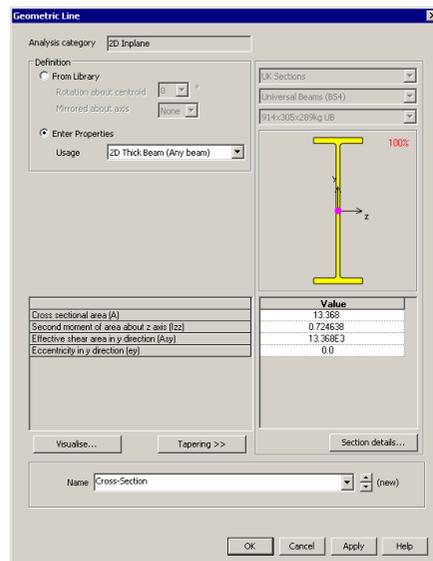
Buttons: OK, Cancel, Apply, Help

### Geometric Properties

The Line features representing the two-span continuous beam are to be assigned with a custom section equivalent to the properties used in the paper by Wang *et al.* In this paper the flexural rigidity EI value of  $2.5E10 \text{ N/m}^2$  and mass per unit length of  $3.4088E4 \text{ kg/m}$  equate to the properties entered below.

Attributes  
 Geometric >  
 Line ...

- Ensure the definition is set to **Enter Properties** for **2D Thick Beam** ( any beam).
- Set the Cross sectional area (A) as **13.368**
- Set the Second moment of area about z axis (Izz) as **0.724638**
- Set the Effective shear area in y direction (Asy) as **13.368E3**
- Ensure the Eccentricity in y direction (ey) is set to **0.0**
- Enter the attribute name to be **Cross-Section** and click **OK**.
- With the lines defining the two-span beam still selected, drag and drop the geometric attribute **Cross-Section** from the  Treeview onto the selected model Lines.



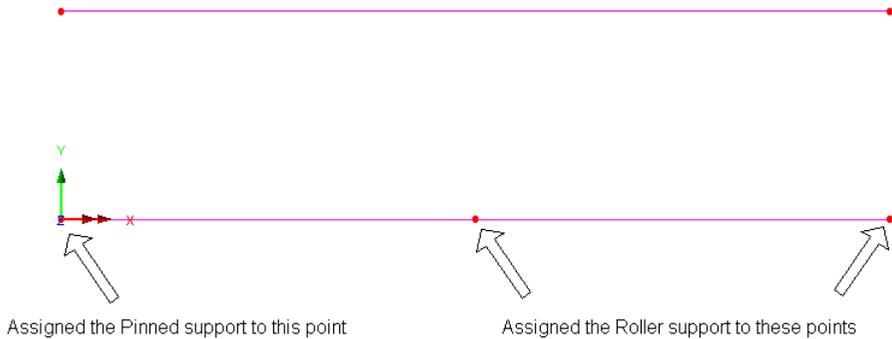
**Note.** The effective shear area has been set to 1000 times the area of the section to effectively remove shear deformation from the calculations of the beams. The paper uses Bernoulli-Euler beams which do not account for shear deformations.

## Supports

The continuous beam is supported with a pinned support at the left-hand end and is supported by roller supports at the middle and right-hand end of the beam.

Attributes  
 Support...

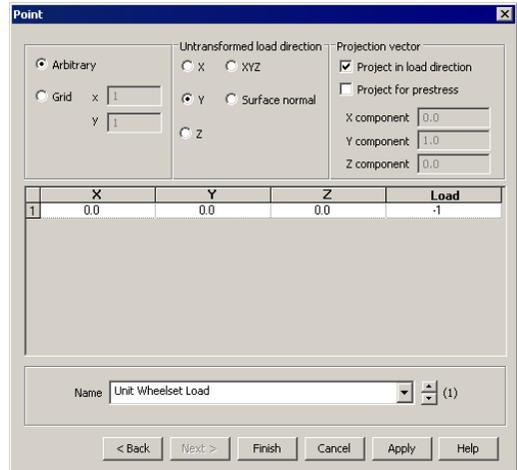
- Set the Translation in X as **Fixed**
- Set the Translation in Y as **Fixed**
- Enter the attribute name as **Pinned** and click **Apply**
- Set the Translation in X as **Free**
- Leave the Translation in Y as **Fixed**
- Enter the attribute name as **Roller** and click **OK**
- Assign the **Pinned support** to the left-hand point, and the **Roller support** to the other two points on the model as indicated below, clicking **OK** to accept the defaults when the Assign Support dialog appears.



## Loading

To allow the IMDPlus moving mass analysis to be carried out a unit load representing the action of the spring-mass system on the beam needs to be included in the model so this can be passed across the beam to calculate the modal forces.

- Click the left-hand mouse button in a blank part of the view window to remove all items from the selection.
- On the Structural Loading dialog, and from the Discrete loading section, select the **Point** loading option and click the **Next** button.
- Ensure that **Arbitrary** is selected
- Ensure that the Untransformed load direction is set to **Y**
- Ensure the **Project in load direction** is selected.
- Enter a single load into the grid with the **X** set to **0** , **Y** set to **0** , **Z** set to **0** and the **Load** set to **-1** so it acts downwards in the global Y axis direction.
- Enter the attribute name as **Unit Wheelset Load** and click **Finish**



## Search Area

When the discrete load representing the single unit wheelset is passed across the structure it is best practice to target the action of this load to the model using a search area.

Attributes

Search Area...

- Set the attribute name as **Continuous Beam** and click **OK**
- Select the lines defining the two-span continuous beam.
- Drag and drop the search area attribute **Continuous Beam** from the  Treeview onto the selected model Lines.
- Click the left-hand mouse button in a blank part of the view window to remove all items from the selection.

## Defining Eigenvalue Controls

Eigenvalue controls are defined as properties of the loadcase.

- In the  Treeview expand **Analysis 1** then right-click on **Loadcase 1** and select **Eigenvalue** from the **Controls** menu.

The Eigenvalue dialog will appear.

The following parameters need to be specified:

- Ensure the Solution is set to **Frequency**
- Set the Eigenvalues required to **Range**
- Ensure the Range specified as is set to **Frequency**
- Set the Minimum frequency as **0**
- Set the Maximum frequency as **50**
- Set the Number of eigenvalues as **0** to solve for all modes in the range
- Ensure the Type of eigensolver is set as **Default**



**Note.** Eigenvalue normalisation is set to **Mass** by default. This is essential if the eigenvectors are to be used for subsequent IMD analyses.

- Click the **OK** button to finish.

File

Save



Save the model file.

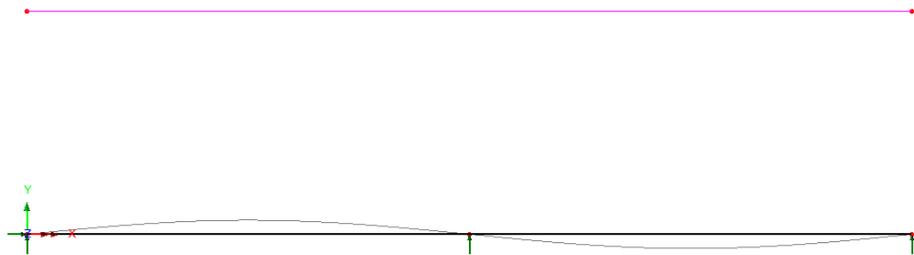
### Running the Analysis

- With the model loaded click the **Solve** button  and the **Solve Now** dialog will be displayed.
- Click the **OK** button to run the analysis.

A LUSAS Datafile will be created from the model information. The LUSAS Solver uses this datafile to perform the analysis.

### Viewing the Results

If the analysis was run from within LUSAS Modeller the results will be loaded on top of the current model and the loadcase results for each eigenvalue can be seen in the  Treeview. Eigenvalue 1 is set to be active by default.



### Checking the Frequencies

The Wang *et al* paper lists only the first two frequencies but the other vertical frequencies can be calculated from the equations in the paper. These frequencies of the vertical modes below 50 Hz of the Bernoulli-Euler beams are listed below:

$$f_1 = 3.363 \text{ Hz}, f_2 = 5.255 \text{ Hz}, f_3 = 13.452 \text{ Hz}, f_4 = 17.025 \text{ Hz}, f_5 = 30.267 \text{ Hz}, f_6 = 35.522 \text{ Hz}$$

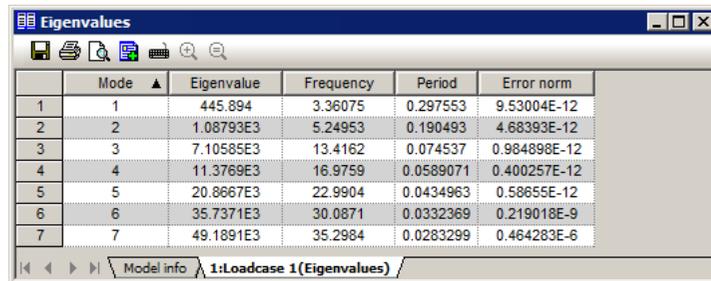
The frequencies obtained from the analysis are obtained from the Print Results Wizard.

- Ensure results type **Eigenvalues** is selected. For loadcase 1, in the available results panel select **Eigenvalues (Frequency)** and click **OK**. The eigenvalue / frequency results will be displayed and a Print Results Wizard entry will be added to the Utilities  treeview.

It can be seen that LUSAS has obtained 7 modes of vibration in the range of 0 to 50 Hz. All of the modes of vibration listed above for the Bernoulli-Euler beam are closely matched but with an additional mode of vibration, mode 5, at 22.990 Hz which was not

Utilities  
Print Results  
Wizard...

captured in the paper. This will be shown to be a horizontal mode of vibration below which is not considered by Wang *et al.*



	Mode ▲	Eigenvalue	Frequency	Period	Error norm
1	1	445.894	3.36075	0.297553	9.53004E-12
2	2	1.08793E3	5.24953	0.190493	4.68393E-12
3	3	7.10585E3	13.4162	0.074537	0.984898E-12
4	4	11.3769E3	16.9759	0.0589071	0.400257E-12
5	5	20.8667E3	22.9904	0.0434963	0.58655E-12
6	6	35.7371E3	30.0871	0.0332369	0.219018E-9
7	7	49.1891E3	35.2984	0.0283299	0.464283E-6

## Checking the Mass Participation Factor



**Note.** In order to carry out a successful IMDPlus analysis you should ensure that a significant proportion of the total mass has been accounted for in the analysis. Ideally this requires checking that around 90% of the total mass has been achieved in the global directions. If less than 90% has been achieved no further modes need be included if the modes of vibration omitted cannot be excited by the dynamic input or a significant proportion of the structure is restrained by support in these directions and therefore cannot participate in the modes of vibration. The acceptability of the included modes of vibration will vary from analysis to analysis but failure to check that a significant proportion of the total mass has been accounted for may lead to important modes being missed and subsequent errors in the analysis results.



**Note.** The mode shapes should also be checked as a mode can have a low mass participation factor and still be significant in the analysis. This often occurs with 'symmetrical modes' where equal mass is moving in opposite directions in the mode shape. This is the case for the first mode in this analysis which is a simple sinusoid shape with one span moving up with the other of equal length and mass moving down. This mode has a zero mass participation factor in the Y-direction but is an important mode in the analysis (see the resonance of the beam to the train speed later in the example). A certain amount of engineering judgement is therefore required for the assessment of the importance of modes of vibration and this should be used in addition to the simple checks on the sum of the mass participation included in the analysis.

### Utilities

Print Results  
Wizard...

- Ensure results type **Eigenvalues** is selected. For loadcase 1, in the available results panel **deselect Eigenvalues (Frequency)**, then select **Sum mass participation factors** and click **OK**. The Sum Mass Participation Factors results will be displayed and another Print Results Wizard entry will be added to the Utilities  treeview.

It can be seen that the 90% value has almost been achieved in both the X- and Y-directions, and this is discussed in the note below.

	Mode	Sum Mass X	Sum Mass Y	Sum Mass Z	Sum Mass THX	Sum Mass THY	Sum Mass THZ	Frequency	Period
1	1	91.3792E-39	36.6168E-27	0.0	0.0	0.0	36.2813E-36	3.36075	0.297553
2	2	11.7668E-36	0.738434	0.0	0.0	0.0	43.1141E-33	5.24953	0.190493
3	3	1.90702E-33	0.738434	0.0	0.0	0.0	43.1146E-33	13.4162	0.074537
4	4	8.16597E-30	0.745294	0.0	0.0	0.0	43.5743E-33	16.9759	0.0589071
5	5	0.810569	0.745294	0.0	0.0	0.0	43.5745E-33	22.9904	0.0434963
6	6	0.810569	0.745294	0.0	0.0	0.0	43.6589E-33	30.0871	0.0332369
7	7	0.810569	0.856006	0.0	0.0	0.0	0.452176E-27	35.2984	0.0283299



**Note.** In this analysis we are only including the first 7 modes of vibration with frequencies up to and including 50 Hz. These 7 modes include the 6 vertical modes considered by Wang *et al* and also an additional horizontal mode, mode 5, with 81.06% mass participation in the X-direction. If more than 7 modes of vibration were included then the sum mass participation would approach the 90% in all directions (solving from 0 to 100 Hz will give 12 modes of vibration which will give 90.06% in the X-direction and 89.97% in the Y-direction but these higher modes may not be excited by the passage of the trainset and may not significantly contribute to the vibration response of the structure).

- Close the results window.



**Note.** To simplify the use of IMDPlus (and provide an alternative to picking IMDPlus menu selections from the Analyses > IMDPlus menu items) for the rest of the example it is recommended that the IMDPlus toolbar is enabled in LUSAS Modeller.

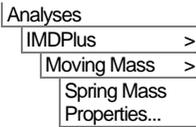
- Select the **View > Toolbars...** menu command, select the **Toolbars** tab, enable the **IMDPlus** option in the list and click **Close**.
- If the IMDPlus toolbar has been enabled, click on the  Moving Mass analysis button in the toolbar to select a moving mass analysis and enable the tool button shortcuts to the dialogs.

## Defining the CRH Trainset

Before the analysis can be performed we need to define the sprung mass representation of the CRH Chinese high speed trainset in LUSAS Modeller. This is a two stage process where

1. the individual spring-mass definitions for the wheelsets / bogies / bodies need to be defined,
2. the complete trainset configuration is defined.

The spring-mass representation from the paper will be used for the analysis which includes only a single spring-mass definition that is used for all of the wheelsets of the trainset.



- Open the IMDPlus Spring-Mass System Properties dialog through the menu or click on the  button in the toolbar. The wheelsets / bodies of the CRH train will be defined with the wheelset as the unsprung mass and the body as the first sprung-mass.

**IMDPlus Spring-Mass System Properties**

Single sprung mass: Single sprung mass

Damping

Nonlinear springs

Polynomial

Piecewise linear

	Unsprung mass	Sprung mass 1
Mass	3300	9500
Elastic spring stiffness		2.5E5
Viscosity coefficient		4.5E4

Spring 1 stiffness

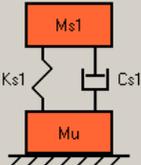
Stiffness	Deformation
1	

Piecewise linear compression

Spring 2 stiffness

Stiffness	Deformation
1	

Piecewise linear tension



Name: CRH Train Wheelset (1)

OK Cancel Apply Help

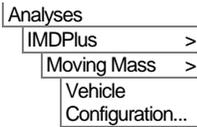
- Select **Single sprung mass** from the drop-down list.
- Ensure **Damping** is selected.
- Enter the Mass of the Unsprung mass as **3300**
- Enter the Mass of Sprung mass 1 as **9500**
- Enter the Elastic spring stiffness of Sprung mass 1 as **2.5E5**
- Enter the Viscosity coefficient of Sprung mass 1 as **4.5E4**
- Enter the Name as **CRH Train Wheelset**
- Click the **OK** button.

## High Speed Train Modelling through Sprung Masses

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The Chinese CRH trainset in the paper consists of a total of eight cars. Using the dimensions shown in the published paper, the following spring-mass systems values are required to define the total trainset.

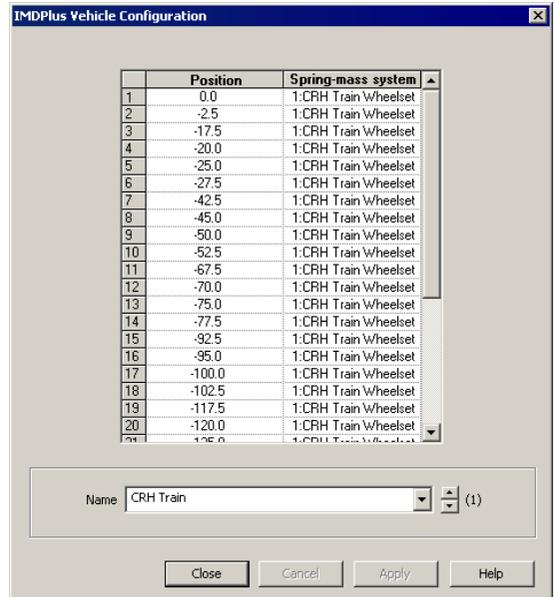
Car	Distance from Previous Axle (m)	Distance from front of train (m)
Locomotive	0	0.0
Locomotive	2.5	-2.5
Locomotive	15	-17.5
Locomotive	2.5	-20.0
Passenger 1	5	-25.0
Passenger 1	2.5	-27.5
Passenger 1	15	-42.5
Passenger 1	2.5	-45.0
Passenger 2	5	-50.0
Passenger 2	2.5	-52.5
Passenger 2	15	-67.5
Passenger 2	2.5	-70.0
Passenger 3	5	-75.0
Passenger 3	2.5	-77.5
Passenger 3	15	-92.5
Passenger 3	2.5	-95.0
Passenger 4	5	-100.0
Passenger 4	2.5	-102.5
Passenger 4	15	-117.5
Passenger 4	2.5	-120.0
Passenger 5	5	-125.0
Passenger 5	2.5	-127.5
Passenger 5	15	-142.5
Passenger 5	2.5	-145.0
Passenger 6	5	-150.0
Passenger 6	2.5	-152.5
Passenger 6	15	-167.5
Passenger 6	2.5	-170.0
Locomotive	5	-175.0
Locomotive	2.5	-177.5
Locomotive	15	-192.5
Locomotive	2.5	-195.0



- Open the IMDPlus Moving Mass Vehicle Configuration dialog through the menu or click on the  button in the toolbar.

The moving mass vehicle is now defined using the Distance from front of train in the table above and the spring-mass system. To simplify the definition of the vehicle the positions and identifiers of the spring-mass systems are included in a Microsoft Excel spreadsheet so they can be copied and pasted into the dialog.

- Locate the **CRH\_Wheelset\_Positions.xls** spreadsheet in the `<<LusasInstallationFolder>>\Examples\Modeller` folder and open it.
- Select all of the 32 positions and spring-mass system identifiers in the first and second columns and copy these to the clipboard using **Ctrl+C**
- Select both the **Position** and **Spring-mass system** headers of the grid in the dialog and paste these into the grid using **Ctrl+V**
- Enter the Name as **CRH Train**



Once finished the dialog should appear as shown.

- Click **OK** to create the vehicle configuration.



**Note.** Alternatively each **Position** in the table can be entered manually and the **Spring-mass system** associated with each of the wheelset/body positions can be selected from the drop-down lists containing the available spring-mass system property attributes.



**Note.** The positions of the wheelsets are (and should be) defined as negative values to ensure that the whole trainset passes over the structure. Negative values place the wheelsets behind the front of the train which is at a position of 0. The movement of the front of the train will be calculated from the start of the path to the end of the path with additional time included in the IMDPlus solution to allow all of the wheelsets with negative positions to pass along the path.

If any wheelsets are defined with positive values then these will already be the distance down the path equivalent to the position entered in the trainset definition. Those wheelsets with positive positions may already be over the structure at the start of the IMDPlus analysis if the start of the path has not been defined to account for this. If any positive positions are used then always ensure that the lines defining the beginning of the path allow sufficient additional length in order to model the transition of the whole of the train onto the structure correctly.



**Note.** Once a trainset or vehicle has been defined it can be saved in a library so it can be imported and used in future analyses without the need to go through all of the definition process above. This is achieved through the **Library Browser** accessed through the **File > Import/Export Model Data...** menu command.

### Importing a Vehicle

If it proves time consuming or you are unable to obtain the same results shown later in the example the spring-mass system and vehicle definition can be imported from the library provided in the `\<LusasInstallationFolder>\Examples\Modeller` folder.



**CRH Train.lmd** contains the vehicle library for the example.

File  
Import/Export Model  
Data...

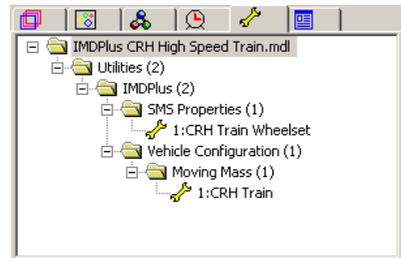
- Select the **Import from library to model** option
- Click on the **Choose file...** button and browse to the **CRH Train.lmd** file in the `\<LusasInstallationFolder>\Examples\Modeller` folder.
- Select the **IMDPlus (2)** entries in the tree
- Click the **OK** button.

### Storage of IMDPlus utilities

After the spring-mass systems and vehicle attributes have been defined they will appear in the Utilities  treeview.



**Note.** If it is required to edit the spring-mass system or vehicle attributes simply double-click with the left mouse button or right-click and choose **Edit Attribute...** on the name of the attribute to bring up the original dialog.



## Moving Mass Analysis

Moving mass calculations are performed using the IMDPlus (Interactive Modal Dynamics) facility. In order to carry out the moving mass analysis of the train travelling across the structure we need to carry out three stages:

1. Define and set-up the path along which the spring-mass systems will travel using a unit load defined as either a discrete point or patch load. For this example the unit wheelset load has already been defined as a discrete point load called **Unit Wheelset Load** which acts vertically
2. Convert the loading along this path from this unit load into modal forces that are applied in the IMDPlus moving mass analysis, to compute the dynamic effects of the spring-mass systems on the response of the structure
3. Run an IMDPlus moving mass analysis to calculate the response of the structure and the spring-mass systems



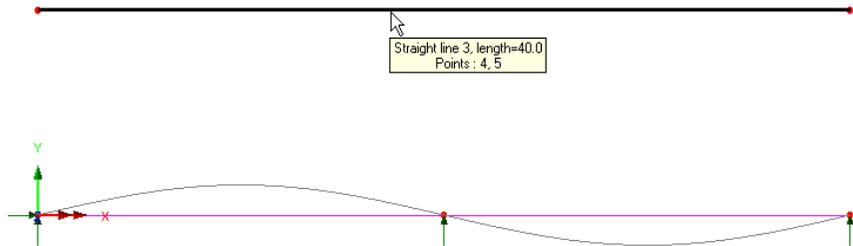
**Note.** This process is similar to the IMDPlus moving load analysis procedure but with the trainset / vehicle defined by the spring-mass systems rather than constant forces.

## Defining the Moving Mass Path

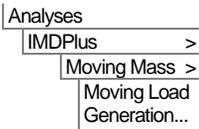
To solve for the passage of the CRH train across the structure the path for the moving mass must be defined. Line 3 (the line representing the path of the moving mass) will be set to be the current selection.

To ensure the whole model is visible:

- In the  Treeview click the right-hand mouse button on the group name **IMDPlus CRH High Speed Train.mdl** and select the **Visible** option from the drop-down menu.
- In the  Treeview ensure the **Geometry** layer is turned on.
- Select the **line** shown (line 3)



**Note.** The path can be defined from multiple lines and arcs but these must form a continuous path without branching.



- Open the IMDPlus Moving Load Generation dialog through the menu or click on the  button in the toolbar.

On startup of the IMDPlus Moving Load Generation dialog, all valid discrete loads and search areas will be made available in the loading options along with information about the path defined by the current selection.

In this example a single discrete load called Unit Wheelset Load which defines the unit loading from a single wheelset of the train is present along with a search area that is assigned to the bridge structure.

- Ensure **1:Unit Wheelset Load** is selected from the Moving load dataset list.
- Ensure **1:Continuous Beam** is selected from the Search area ID for the assignment of the discrete loading.
- Ensure **Project onto line** is selected (the analysis is 2D so this should be the only option).
- Click on the **Advanced** button to adjust the inclusion of load characteristics. On the Moving Load Advanced Options dialog choose the **Exclude all load** option for loads outside the search area, ensure the load factor is **1.0** and click the **OK** button.

- On the IMDPlus Moving Load Generation dialog set the Incremental distance to **0.1**



**Note.** Using search areas targets the application of the loading to the required features as described in the Modeller Reference Manual.



**Note.** By default the incremental distance is set to one tenth of the length of the line along which the load moves.

- Leave all other settings as their defaults and click the **OK** button to proceed and choose **Yes** to save the current model.

The program will now generate the loading information for the 401 locations of the unit wheelset along the path before returning control back to LUSAS Modeller.

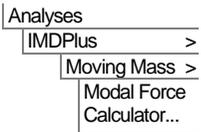


**Note.** The discrete loading locations defined by this dialog will be tabulated into five datafiles with a maximum of 100 loadcases each and the analyses will be performed automatically. These analyses will use the same file basename as the original model with a numeric indicator appended to it (e.g. \_00001, \_00002, etc). They are required for the modal force calculation stage.

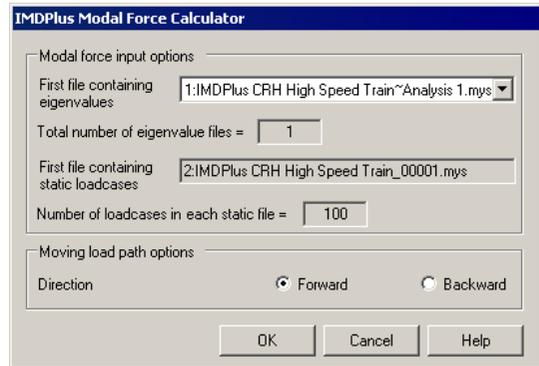
## Generating the Modal Force History for the Moving Mass

In the previous stage the passage of the unit train wheelset across the structure has been defined. The modal forces for the IMDPlus solution now need to be calculated using the Modal Force Calculator.

## High Speed Train Modelling through Sprung Masses

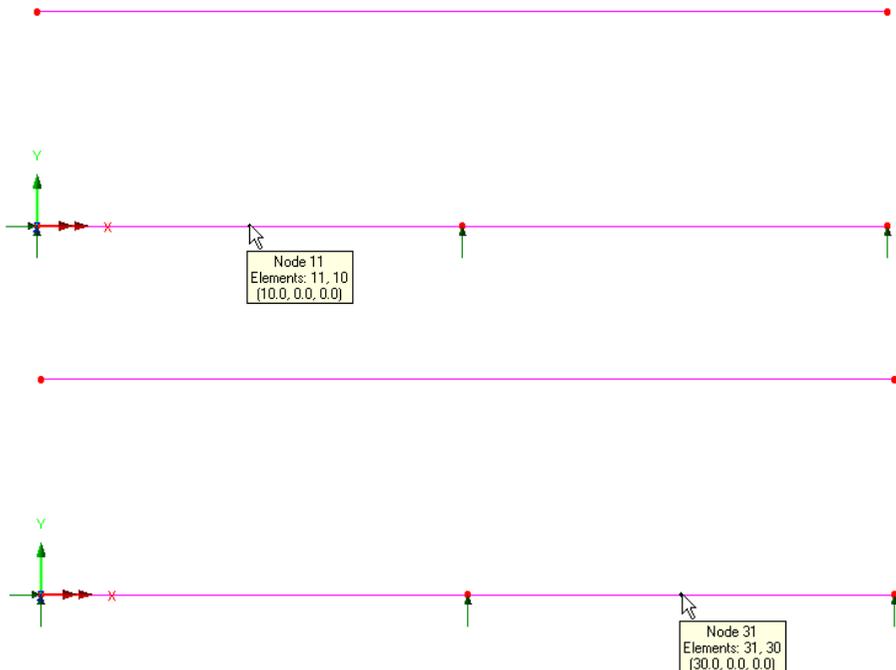


- Open the IMDPlus Modal Force Calculator dialog through the menu or click on the  button in the toolbar.
- Click the **OK** button to accept the default information and proceed.



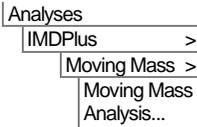
### Identification of Output Nodes

In this example all of the output relates to the mid-spans of the two-span continuous beam model. If the model is identical then these node numbers should be node 11 for the first span and node 31 for the second span as shown below. These nodes will be entered into the IMDPlus dialogs later.



## Defining the Moving Mass Parameters

All of the basic moving mass information has now been defined for the IMDPlus analysis. The next stage is to define the included modes, damping and speed parameters.



- Open the IMDPlus Moving Mass Analysis Control dialog through the menu or click on the  button in the toolbar.

- Ensure the Vehicle configuration contains **1:CRH Train**

- Ensure that the **Path surface profile** is not selected.

- Ensure that Included modes is set to **All**. If this is not the case, click on the **Modify...** button and ensure the **All modes** option is selected.

- Click on the **Modify...** button to change the Default damping. Enter **2.5** for the default damping and click the **OK** button.

- In the Solution control section, click on the **Modify...** button to change the advanced solution options. Click the **Defaults** button to set the default options and click the **OK** button.

- Enter the Minimum Speed as **69.44**, the Maximum Speed as **69.44** and the Speed Increment as **0** to analyse a single speed of 69.44 m/s which is equivalent to 250 kph.

- Enter the Quiet time after passage of load as **5**

- Deselect the **IMDPlus determining time step (Nyquist)** option so we can specify the required time step.

- Enter the Solution time step as **2E-3**

- Click the **Next** button to proceed. When prompted about missing total mass due to mass participation factors below 90% choose **Yes** to continue. For this analysis we are only interested in the contributions of modes of vibration up to and including the first 7 modes (0 to 50 Hz) which means that we are not going to achieve the 90% total mass target.

The IMDPlus Moving Mass Output Control dialog will appear.

### Vertical Mid-Span Displacement Responses

The IMDPlus Output Control dialog controls the results output for the model. The vertical displacement response at the mid-span of the two spans of the continuous beam, nodes 11 and 31, for a speed of 250 kph will be investigated. The first span, node 11 is investigated first.

- Choose **Node** and select Extent as **Individual**
- Enter Node number **11** (This is the node in the mid-span of the first span)
- Select **Displacement** results of **DY**
- Ensure **Individual items** is selected. **Sum of chosen items** and **Average of chosen items** will not be available as an individual node is being processed.
- Ensure **Response time history** is selected.
- Ensure **Generate graphs in Modeller** is selected.
- Click the **Apply** to proceed with the analysis.

**IMDPlus Moving Mass Output Control**

Node/element selection

Node  Element Entity **Displacement**

Extent **Individual** Component **DY**

Node **11**

Individual items  
 Sum of chosen items  
 Average of chosen items

Spring-mass system selection

Spring-mass system Result **Unsprung mass**

Extent **Individual** Entity **Displacement**

System **1 (0.0) CRH Train**

Output control

Response time history

Peak response summary  Positive/negative  Envelope  
 Absolute

Secondary response spectra (SRS)

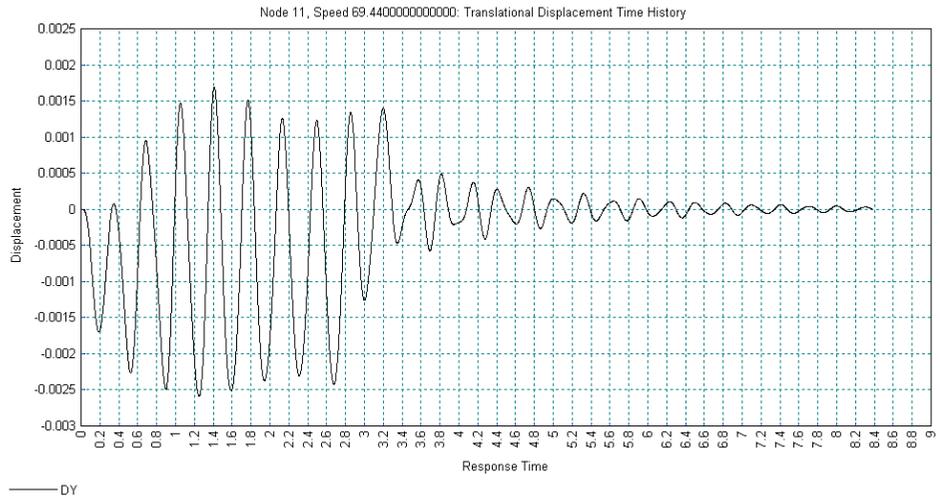
Modal combination/factor history

Generate graphs in Modeller Speed to graph **1:63.44**

Generate textfile output



**Note.** Clicking the Apply button instead of the Finish button keeps the IMDPlus session active for subsequent graph plotting.



The graph generated shows the vertical displacement response at the mid-span of the first span of the continuous beam for the passage of the CRH trainset at 250 kph. The peak downwards deflection of the beam is approximately 2.6 mm and the peak upwards deflection approximately 1.7 mm. The quiet time after the passage of the trainset allows the decay of the response to be observed with most of the dynamic response damped out at around 8.5 seconds after the trainset first enters the continuous beam. The response matches the response of the first span in the Wang *et al* paper, noting that the sign conventions are opposite to the paper which has a positive vertical displacement in the downwards direction.

- When the graph has been viewed, close the graph

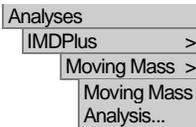
The vertical displacement response at the mid-span of the second span of the continuous beam for the speed of 250 kph will now be investigated.

If the IMDPlus toolbar has been enabled,

- Click on the  button in the toolbar to open the IMDPlus Output Control dialog

If the IMDPlus toolbar is not enabled,

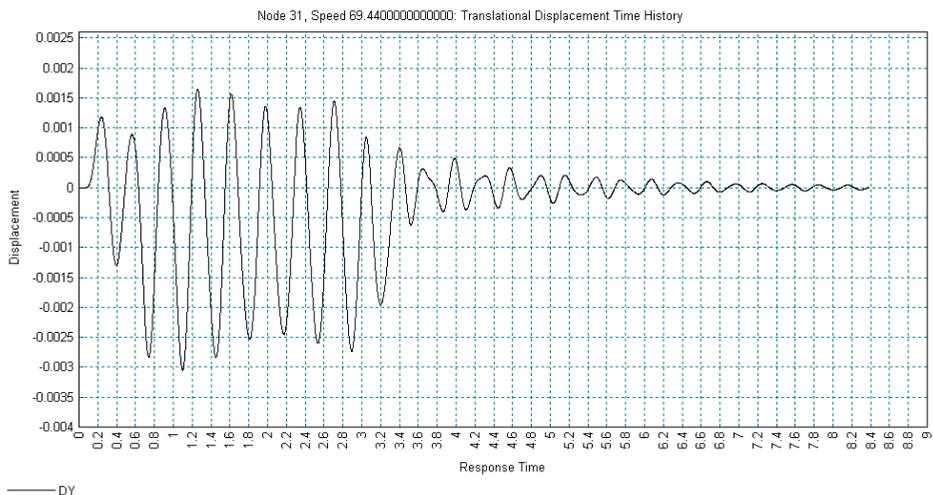
- Open the IMDPlus Moving Mass Analysis Control dialog and click **Next >** to keep the existing analysis control settings and open the IMDPlus Output Control dialog.
- Ensure **Node** and Extent as **Individual** are still selected
- Enter Node number **31** (This is the node in the mid-span of the second span)
- Ensure you are plotting **Displacement** results of **DY** and make sure all other dialog input is unchanged



- Click the **Finish** to proceed.

The IMDPlus will now proceed for the second span. The solution time for this analysis will be quicker than for the first node because of the presence of binary results databases that are saved to the disk.

- Click **No** when asked whether to free up disk space by deleting the temporary files created by IMDPlus. Choosing no will save data extraction times for subsequent analyses at the cost of some disk space.

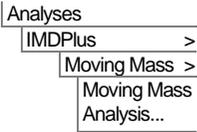


The graph generated shows the vertical displacement response at the mid-span of the second span of the continuous beam for the passage of the CRH trainset at 250 kph. The peak downwards deflection of the beam is approximately 3 mm and the peak upwards deflection approximately 1.6 mm. The response again matches the response of the second span in the Wang *et al* paper, noting that the sign conventions are opposite to the paper which has a positive vertical displacement in the downwards direction.

- When the graph has been viewed, close the graph

## Peak Vertical Displacement of Mid-Spans for Multiple Speeds

The response of the two spans of the continuous beam were investigated for a single speed of 250 kph. The response of these two spans for a range of speeds between 25 kph and 600 kph (in 25 kph increments) is now investigated to determine the worst speeds for the response of the beam.



- Open the IMDPlus Moving Mass Analysis Control dialog through the menu or click on the  button in the toolbar.

- Ensure that the Moving mass input and Mode/damping control sections of the dialog are unchanged from the previous analyses

- Enter the Minimum Speed as **6.944**, the Maximum Speed as **166.667** and the Speed Increment as **6.944** to analyse a range of speeds between 6.944 m/s (25 kph) and 166.667 m/s (600 kph) in increments of 6.944 m/s (25 kph).

- Ensure the Quiet time after passage of load is **5**

- Ensure the **IMDPlus determining time step (Nyquist)** option is deselected and the Solution time step is **2E-3**

- Click the **Next** button to proceed. When asked whether the speed range is correct choose **Yes** to continue. When prompted about missing total mass due to mass participation factors below 90% choose **Yes** to continue.

The IMDPlus Moving Mass Output Control dialog will appear.

- Ensure **Node** and Extent as **Individual** are still selected
- Enter Node number **11**
- Ensure **Displacement** results of **DY** is selected.
- Ensure **Individual items** is selected.
- Deselect **Response time history**.
- Select **Peak response summary** and **Absolute**
- Ensure **Generate graphs in Modeller** is selected.
- Click the **Apply** to proceed.

**IMDPlus Moving Mass Output Control**

Node/element selection

Node  Element Entity: Displacement

Extent: Individual Component: DY

Node: 11

Individual items  
 Sum of chosen items  
 Average of chosen items

Spring-mass system selection

Spring-mass system Result: Unsprung mass

Extent: Individual Entity: Displacement

System: 1 [0.0] CRH Train V

Output control

Response time history

Peak response summary  Positive/negative  Envelope  
 Absolute

Secondary response spectra (SRS)

Modal combination/factor history

Generate graphs in Modeller Speed to graph: 1:6.944

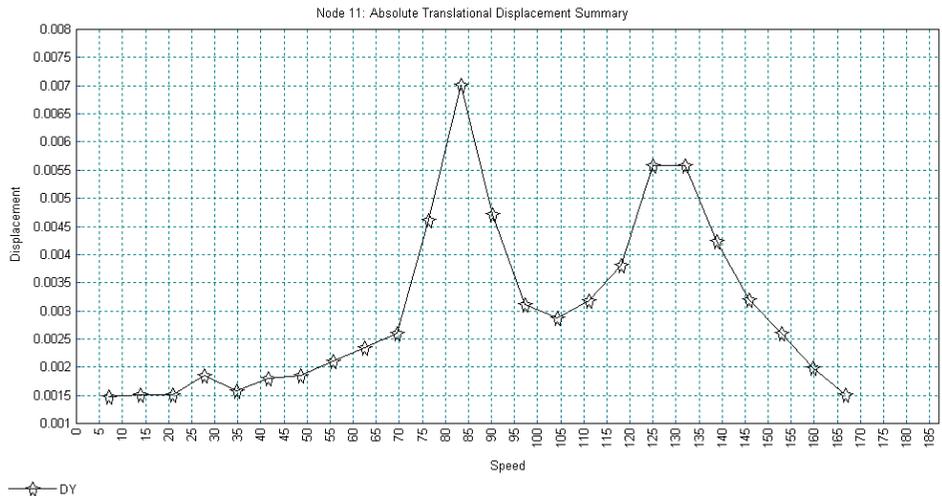
Generate textfile output

< Back Finish Apply Cancel Help

IMDPlus will now calculate the sprung-mass and continuous beam interaction behaviours for all 24 of the speeds chosen in the 25 kph to 600 kph range.



**Note.** Due to the complexity of the equilibrium calculations of the compatible responses of the 32 spring-mass systems and the continuous beam as they pass along the path for the 24 speeds, the first analysis will take extra time to complete. Once this is done, so long as no changes to the modal information or analysis options are made, all subsequent solutions will be considerably faster due to the presence of binary results databases on disk.



The graph shows that the worst vertical displacement response of the first span occurs around 83.328 m/s which is equivalent to a train speed of 300 kph. At this speed the peak vertical displacement response is approximately 7 mm. A second peak is observed between speeds of 124.992 m/s (450 kph) and 131.936 m/s (475 kph). Both of these peaks agree with the Wang *et al* paper which states that the speeds where resonance is induced in the beam for the first two modes of vibration are 302.67 kph and 472.98 kph. The magnitudes of the two peaks are also in agreement with Wang *et al*.

- When the graph has been viewed, close the graph

The peak vertical displacement response at the mid-span of the second span of the continuous beam for the same speed range will now be investigated.

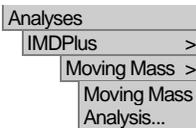
If the IMDPlus toolbar has been enabled,

- Click on the  button in the toolbar to open the IMDPlus Output Control dialog

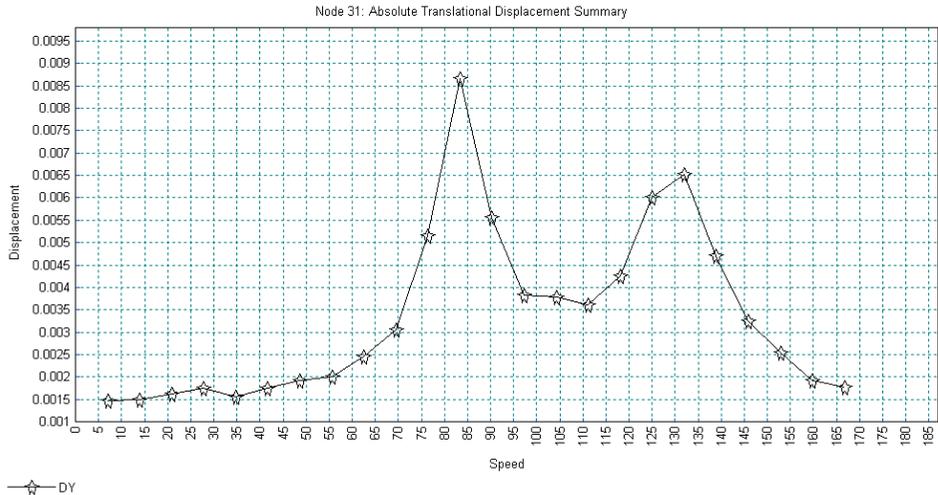
If the IMDPlus toolbar is not enabled,

- Open the IMDPlus Moving Mass Analysis Control dialog and click **Next >** to keep the existing analysis control settings and open the IMDPlus Output Control dialog.

- Ensure **Node** and Extent as **Individual** are still selected
- Enter Node number **31**
- Ensure you are plotting **Displacement** results of **DY** and make sure all other dialog input is unchanged
- Click the **Apply** to proceed.



The IMDPlus analysis will now proceed for the second span. The solution time for this analysis over the range of speeds will be much quicker than for the first node because of the presence of binary results databases that are saved to the disk.

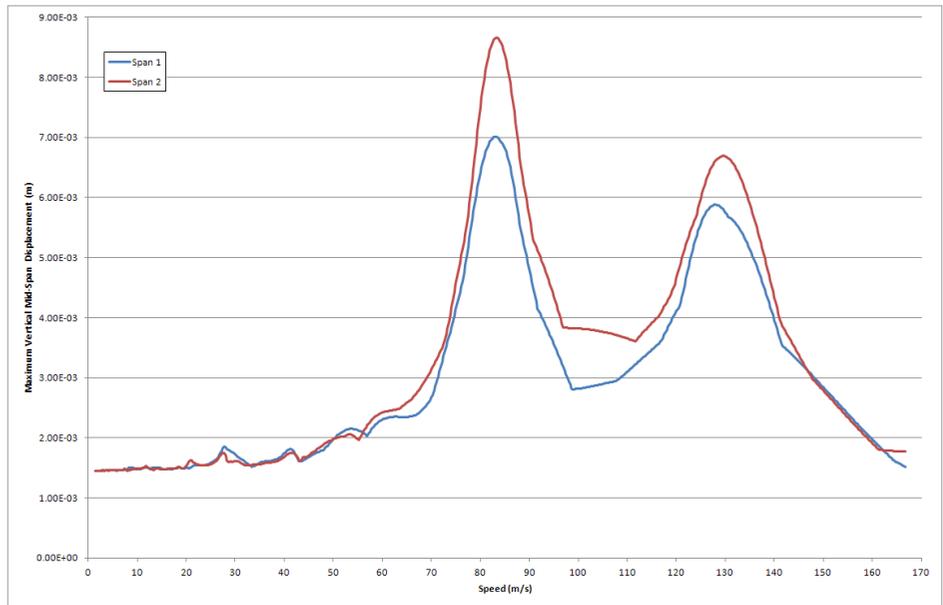


The graph shows that the worst vertical displacement response of the second span occurs again around 83.328 m/s which is equivalent to a train speed of 300 kph. At this speed the peak vertical displacement response is approximately 8.7 mm. A second peak is again observed between speeds of 124.992 m/s (450 kph) and 131.936 m/s (475 kph). Both of these peaks agree with the Wang *et al* paper and a more refined speed range would provide the same smooth curves in the paper.

- When the graph has been viewed, close the graph



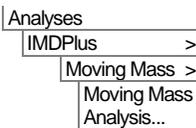
**Note.** The Wang *et al* paper considered speeds between 5 kph and 600 kph in very small increments and therefore appears much smoother than the two graphs above. If this analysis is conducted with a similar resolution of speed increment then the following graphs are obtained from IMDPlus which agree with those in the paper (plotted together in Microsoft Excel for a minimum speed of 5 kph, a maximum speed of 600 kph and a speed increment of 1 kph).



## Resonant Response of Mid-Spans at 300 kph

The train speed of 300 kph (83.328 m/s) showed that there was a resonant response in the continuous beam at around this speed. The time history response of the two mid-span nodes of the continuous beam will now be investigated at this speed.

- Click on a blank part of the model window to deselect the line (if it is still selected)
- Right click on a blank part of the model window and choose **Advanced Selection...**
- Set the Type and name as **Node** and enter **11,31** in the textbox
- Click on **OK** to select both mid-span nodes
- Open the IMDPlus Moving Mass Analysis Control dialog through the menu or click on the  button in the toolbar.



- Ensure that the Moving mass input and Mode/damping control sections of the dialog are unchanged from the previous analyses
- Enter the Minimum Speed as **83.328**, the Maximum Speed as **83.328** and the Speed Increment as **0** to analyse a single speed of 83.328 m/s which is equivalent to 300 kph.
- Ensure the Quiet time after passage of load is **5**
- Ensure the **IMDPlus determining time step (Nyquist)** option is deselected and the Solution time step is **2E-3**
- Click the **Next** button to proceed and click **Yes** when informed about the analysis having a mass participation below 90%.

**IMDPlus Moving Mass Analysis Control**

Moving mass input

Vehicle configuration: 1:CRH Train

Path surface profile: [Browse...]

Mode/damping control

Included modes: All [Modify...]

Default damping: 2.5 % [Modify...]

Total participating mass

X-Direction: 81.05 %

Y-Direction: 85.60 %

Z-Direction: 0.00 %

Solution control

Advanced solution options: [Modify...]

Path surface profile options: [Modify...]

Minimum speed: 83.328

Maximum speed: 83.328

Speed increment: 0

Quiet time after passage of load: 5

IMDPlus automatic time step (Nyquist)

Solution time step: 2E-3

< Back   Next >   Cancel   Help

- Choose **Node** and Extent as **Selected**
- Ensure **Displacement** results of **DY** is selected
- Ensure **Individual items** is selected
- Ensure **Sum of chosen items** and **Average of chosen items** are deselected
- Select **Response time history**
- Deselect **Peak response summary**
- Ensure **Generate graphs in Modeller** is selected
- Ensure the Speed to graph is set to **1:83.328** (equivalent to 300 kph)
- Click the **Apply** to proceed
- Choose **Yes** when asked whether you wish to process all selected nodes

**IMDPlus Moving Mass Output Control**

Node/element selection

Node  Element Entity: Displacement

Extent: Selected Component: DY

Node: 31

Individual items  
 Sum of chosen items  
 Average of chosen items

Spring-mass system selection

Spring-mass system Result: Unsprung mass

Extent: Individual Entity: Displacement

System: 1 (0.0) CRH Train

Output control

Response time history  
 Peak response summary  Positive/negative  Envelope  
 Absolute

Secondary response spectra (SRS)

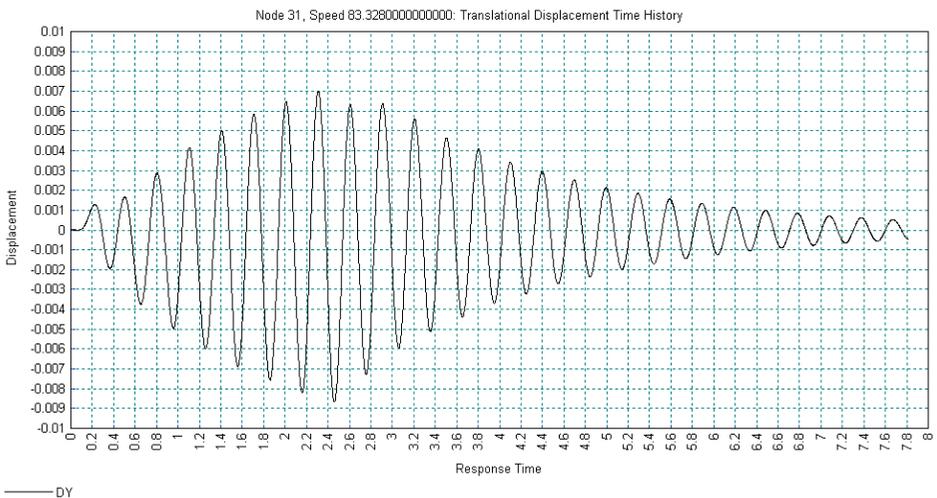
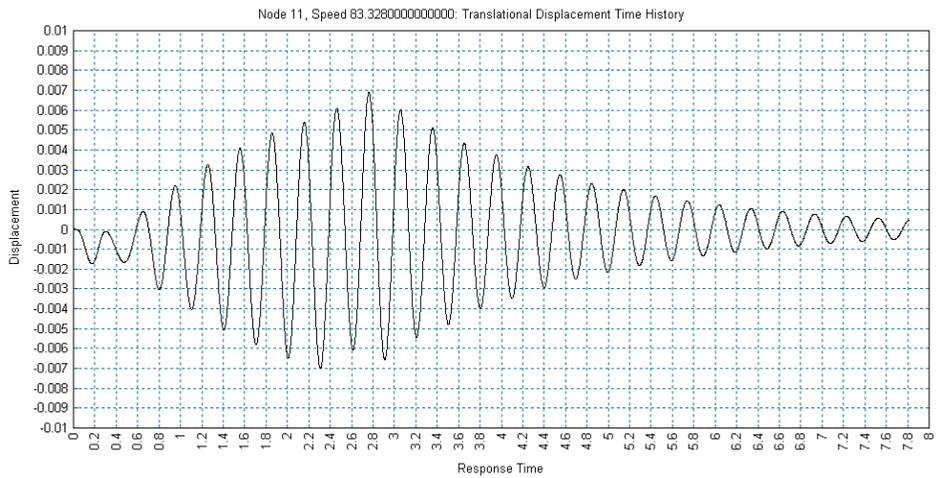
Modal combination/factor history

Generate graphs in Modeller Speed to graph: 1:83.328

Generate textfile output

< Back Finish Apply Cancel Help

The responses of the two mid-span nodes are shown below, firstly for the first span (node 11) and secondly for the second span (node 31). In both figures the resonance of the continuous beam is observed until the departure of the last wheelset of the train from the structure after 2.82 seconds. After this time the damping causes the decay of the vertical response of the beam which will eventually return to a static state.



- When the graphs have been displayed, close the graphs

### Displacement Time Histories of First and Last Spring-Mass Systems

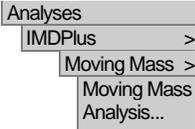
In the previous analyses attention was focused on the vertical displacement response of the continuous beam from the passage of the CRH trainset over a range of speeds. The response of the actual train is now investigated.

If the IMDPlus toolbar has been enabled,

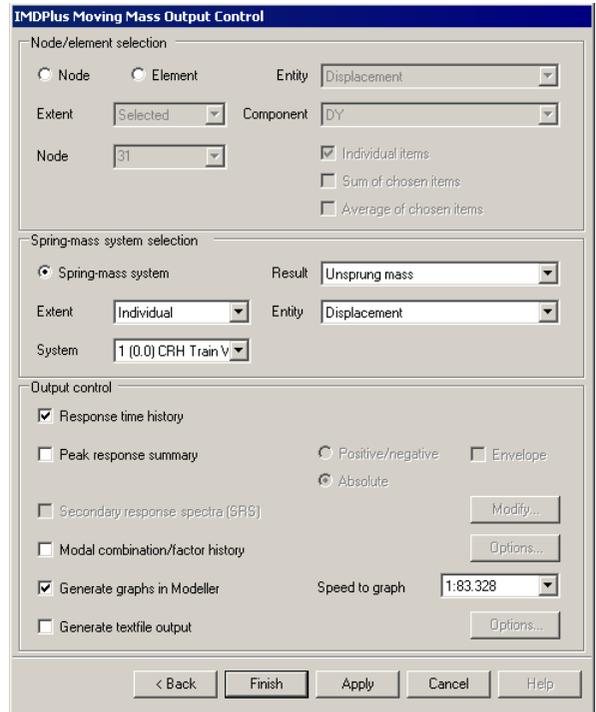
- Click on the  button in the toolbar to open the IMDPlus Output Control dialog

If the IMDPlus toolbar is not enabled,

- Open the IMDPlus Moving Mass Analysis Control dialog and click **Next >** to keep the existing analysis control settings and open the IMDPlus Output Control dialog.

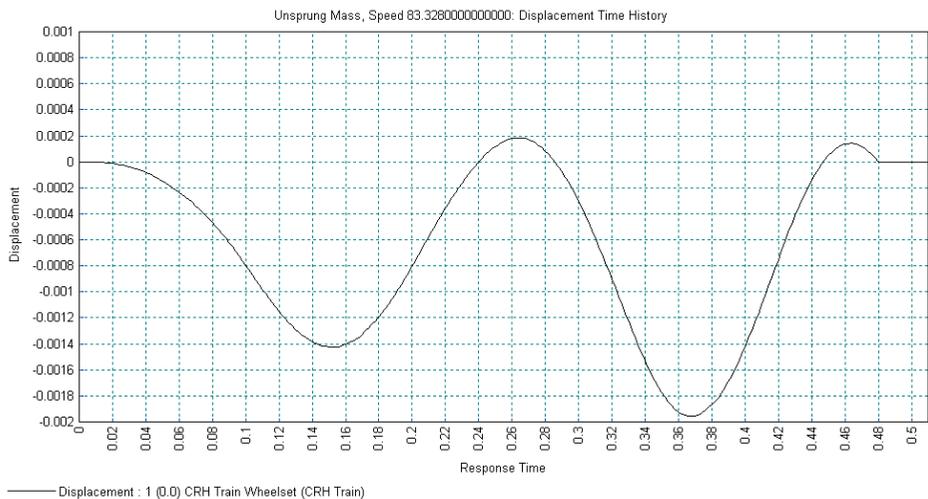
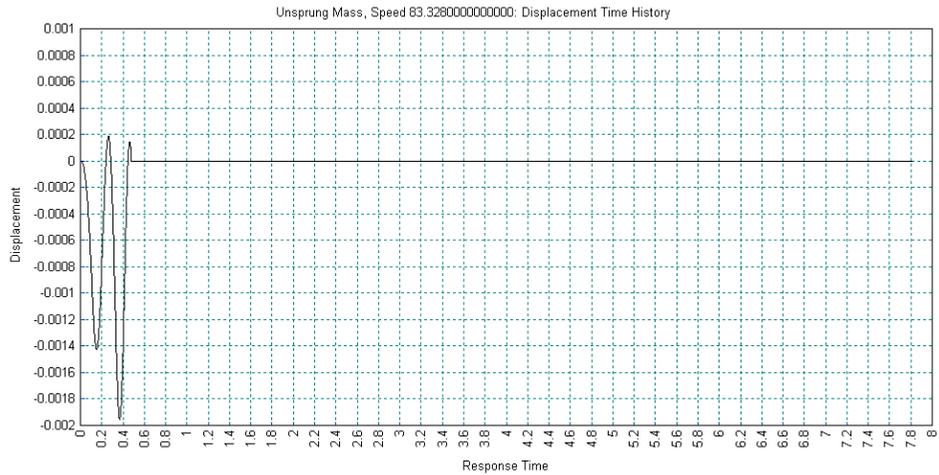


- Select **Spring-mass system** and Extent as **Individual**
- Set the System to **1 (0.0) CRH Train Wheelset**
- Set the Result to **Unsprung mass**
- Set the Entity to **Displacement**
- Ensure all of the selections in the Output control are identical to the previous analysis
- Click the **Apply** to proceed.



The plotting of the results will now proceed. IMDPlus should not need to resolve the analysis since the spring-mass system results should already be available.

## High Speed Train Modelling through Sprung Masses



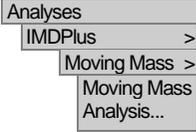
The second graph shows the curve after zooming into the first 0.5 seconds of the time history. This indicates that the displacement of the unsprung mass returns to zero (i.e. the first spring-mass system has left the structure) after about 0.48 seconds. This can be confirmed by dividing the total length of the continuous beam by the speed,  $40 / 83.328 = 0.48$  seconds.

- When the graph has been viewed, close the graph

If the IMDPlus toolbar has been enabled,

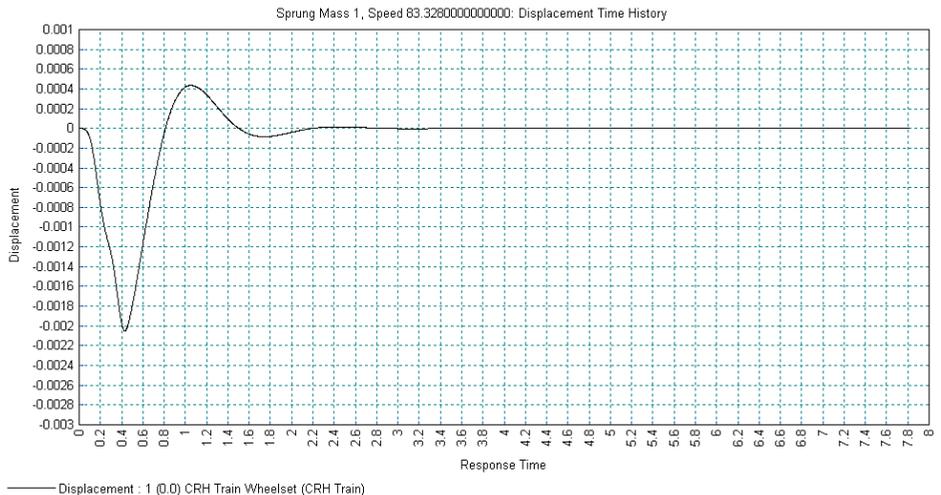
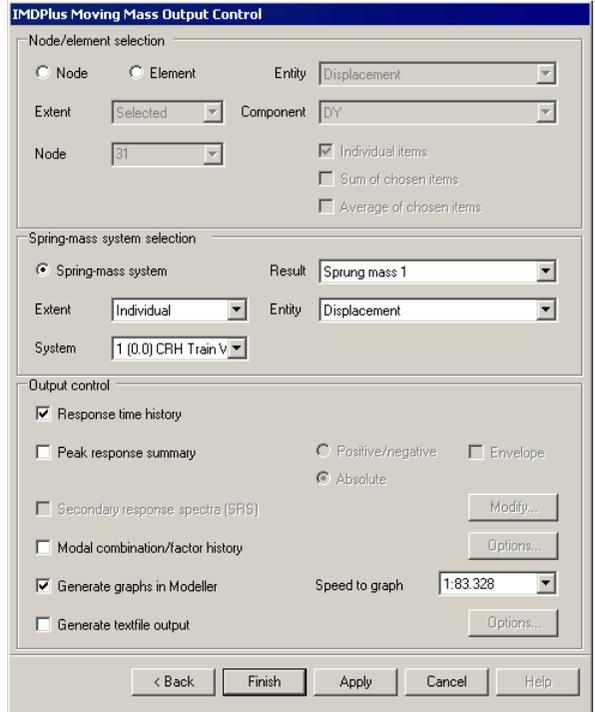
- Click on the  button in the toolbar to open the IMDPlus Output Control dialog

If the IMDPlus toolbar is not enabled,



- Open the IMDPlus Moving Mass Analysis Control dialog and click **Next >** to keep the existing analysis control settings and open the IMDPlus Output Control dialog.

- Ensure **Spring-mass system** is selected.
- Ensure Extent is set to **Individual**
- Ensure the System is set **1 (0.0) CRH Train Wheelset**
- Set the Result to **Sprung mass 1**
- Ensure the Entity is set to **Displacement**
- Ensure all of the selections in the **Output control** are identical to the previous analysis.
- Click the **Apply** to proceed.



The graph shows that the sprung mass representing the body of the first wheelset of the train displaces just over 2mm from its static position 0.43 seconds after it first enters the continuous beam and the viscous damping of the spring-mass system quickly damps out the body motion within 2 to 2.5 seconds after the wheelset leaves the structure (which occurs at 0.48 seconds).

- When the graph has been viewed, close the graph



**Note.** All results for the sprung masses are with respect to their static equilibrium positions. Static displacements of the sprung masses are summarised in the IMDPlus.log file at the beginning of the analysis.

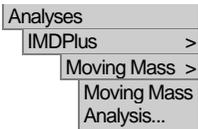
The last spring-mass system is 195m behind the first one and therefore the continuous beam will have received dynamic input from 31 spring-mass systems by the time it enters the beam. As a result, the vertical displacement of the unsprung and sprung masses are expected to be different from the graphs presented above.

If the IMDPlus toolbar has been enabled,

- Click on the  button in the toolbar to open the IMDPlus Output Control dialog

If the IMDPlus toolbar is not enabled,

- Open the IMDPlus Moving Mass Analysis Control dialog and click **Next >** to keep the existing analysis control settings and open the IMDPlus Output Control dialog.



- Ensure **Spring-mass system** is selected
- Ensure Extent is set to **Individual**
- Set the System to **32 (-195.0) CRH Train Wheelset**
- Set the Result to **Unsprung mass**
- Ensure the Entity is set to **Displacement**
- Ensure all of the selections in the **Output control** are identical to the previous analysis
- Click the **Apply** to proceed.

**IMDPlus Moving Mass Output Control**

Node/element selection

Node  Element Entity: Displacement

Extent: Selected Component: DY

Node: 31

Individual items  
 Sum of chosen items  
 Average of chosen items

Spring-mass system selection

Spring-mass system Result: Unsprung mass

Extent: Individual Entity: Displacement

System: 32 (-195.0) CRH Tr.

Output control

Response time history

Peak response summary  Positive/negative  Envelope

Secondary response spectra (SRS)  Absolute

Modal combination/factor history

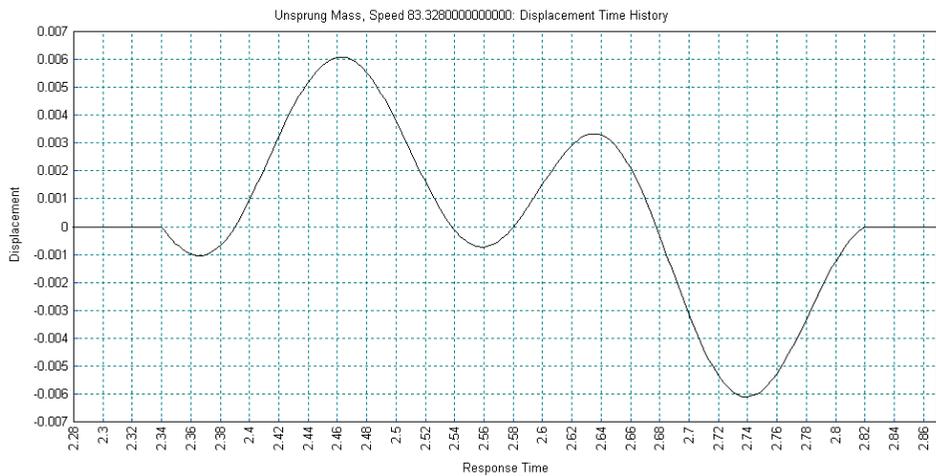
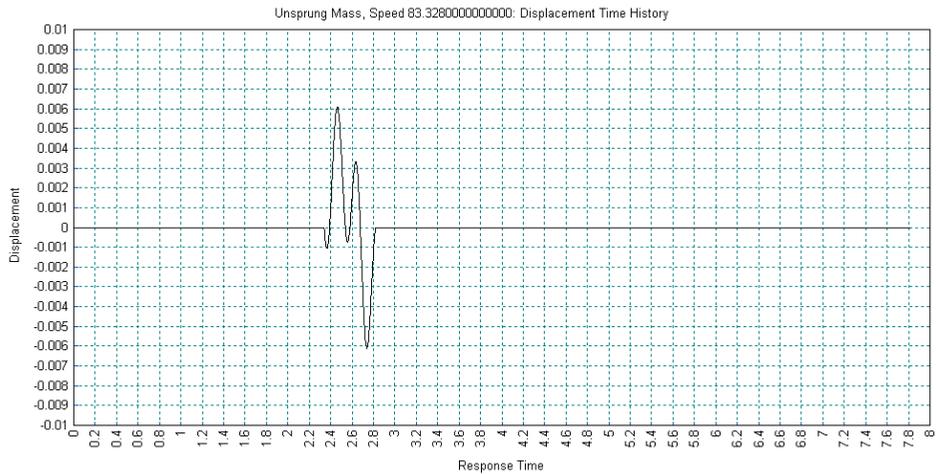
Generate graphs in Modeller Speed to graph: 1:83.328

Generate textfile output

< Back Finish Apply Cancel Help

The graphs show the vertical displacement of the last sprung mass for the whole time history and having zoomed into the region where the dynamic behaviour is observed. This indicates that the displacement of the unsprung mass is zero until it enters the structure at about 2.34 seconds and returns to zero after it has left the structure at about 2.8 seconds. Comparison of the vertical displacement response while the spring-mass system is over the beam with the first unsprung mass system above shows how the existing vibration of the continuous beam has altered the unsprung behaviour.

## High Speed Train Modelling through Sprung Masses



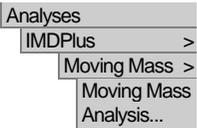
- When the graph has been viewed, close the graph

If the IMDPlus toolbar has been enabled,

- Click on the  button in the toolbar to open the IMDPlus Output Control dialog

If the IMDPlus toolbar is not enabled,

- Open the IMDPlus Moving Mass Analysis Control dialog and click **Next >** to keep the existing analysis control settings and open the IMDPlus Output Control dialog.



- Ensure **Spring-mass system** is selected
- Ensure Extent is set to **Individual**
- Ensure the System is set **32 (-195.0) CRH Train Wheelset**
- Set the Result to **Sprung mass 1**
- Ensure the Entity is set to **Displacement**
- Ensure all of the selections in the **Output control** are identical to the previous analysis
- Click the **Apply** to proceed.

**IMDPlus Moving Mass Output Control**

Node/element selection

Node  Element Entity: Displacement

Extent: Selected Component: DY

Node: 31

Individual items  
 Sum of chosen items  
 Average of chosen items

Spring-mass system selection

Spring-mass system Result: Sprung mass 1

Extent: Individual Entity: Displacement

System: 32 (-195.0) CRH Tr.

Output control

Response time history

Peak response summary  Positive/negative  Envelope  
 Absolute

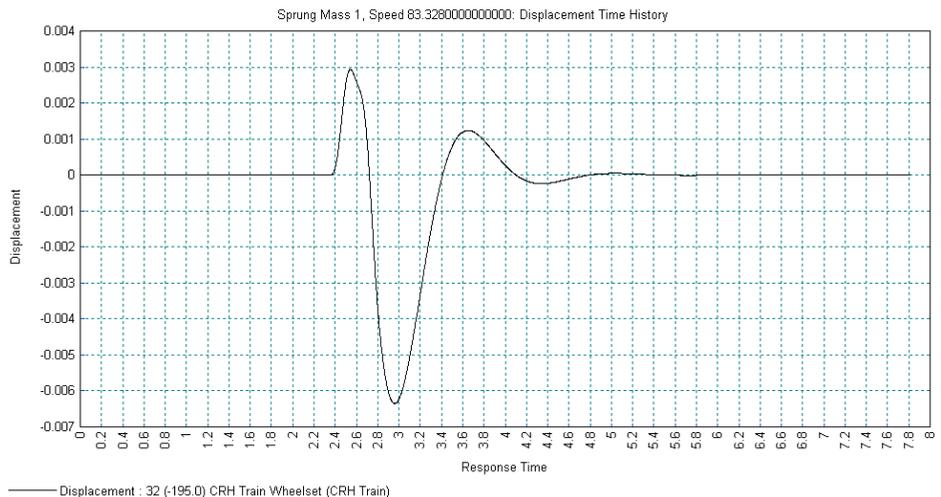
Secondary response spectra (SRS)

Modal combination/factor history

Generate graphs in Modeller Speed to graph: 1.83.328

Generate textfile output

< Back Finish Apply Cancel Help



The graph shows that the sprung mass representing the body of the last wheelset of the train displaces almost 6.5 mm from its static position while it is over the continuous beam before the damping of the spring-mass system quickly damps out the body motion after the wheelset leaves the structure.

- When the graph has been viewed, close the graph

### Acceleration Time Histories of First and Last Spring-Mass System Bodies

The acceleration response of the sprung masses representing the bodies of train locomotives and passenger cars can be used to assess the comfort level that would be experienced by passengers. The accelerations of the sprung mass for the first and last sprung mass systems will be investigated.

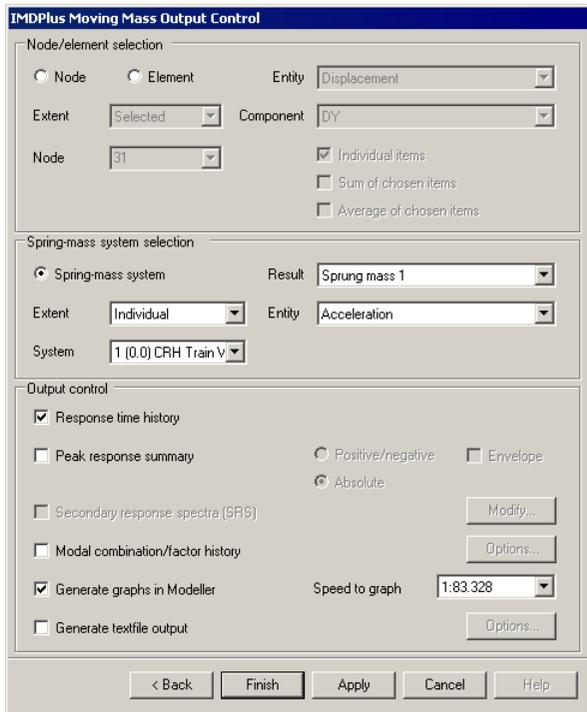
If the IMDPlus toolbar has been enabled,

- Click on the  button in the toolbar to open the IMDPlus Output Control dialog

If the IMDPlus toolbar is not enabled,

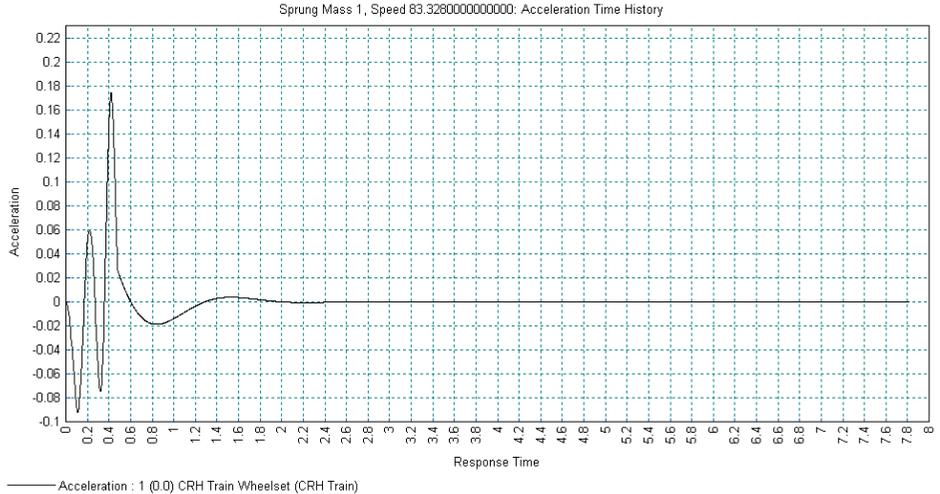
- Open the IMDPlus Moving Mass Analysis Control dialog and click **Next >** to keep the existing analysis control settings and open the IMDPlus Output Control dialog.

- Ensure **Spring-mass system** is selected
- Ensure Extent is set to **Individual**
- Set the System to **1 (0.0) CRH Train Wheelset**
- Ensure the Result is set to **Sprung mass 1**
- Set the Entity to **Acceleration**
- Ensure all of the selections in the **Output control** are identical to the previous analysis
- Click the **Apply** to proceed.



The graph shows the vertical acceleration response of the body of the first wheelset which experiences a maximum acceleration of  $+0.174 \text{ m/s}^2$  or  $0.018 \text{ g}$  as it is about to

leave the continuous beam. This peak acceleration for the first wheelset occurs while the dynamic response of the continuous beam is still increasing and therefore a higher peak acceleration might be expected for subsequent wheelsets of the train.



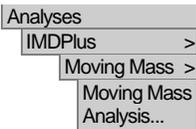
- When the graph has been viewed, close the graph

If the IMDPlus toolbar has been enabled,

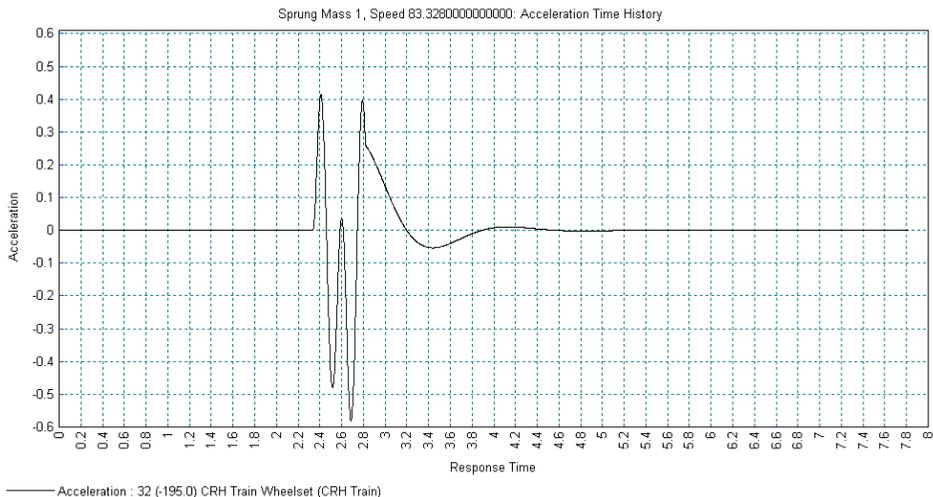
- Click on the  button in the toolbar to open the IMDPlus Output Control dialog

If the IMDPlus toolbar is not enabled,

- Open the IMDPlus Moving Mass Analysis Control dialog and click **Next >** to keep the existing analysis control settings and open the IMDPlus Output Control dialog.



- Ensure **Spring-mass system** is selected
- Ensure Extent is set to **Individual**
- Set the System to **32 (-195.0) CRH Train Wheelset**
- Ensure the Result is set to **Sprung mass 1**
- Ensure the Entity is set to **Acceleration**
- Ensure all of the selections in the **Output control** are identical to the previous analysis
- Click the **Finish** to proceed. Click **No** when asked whether to free up disk space.



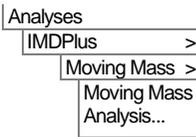
The graph shows the vertical acceleration response of the body of the last wheelset and indicates that it experiences a maximum acceleration of  $-0.582 \text{ m/s}^2$  or  $-0.059 \text{ g}$  while it is about to leave the continuous beam. This peak acceleration for the last wheelset is significantly higher than the first wheelset as might be expected for subsequent

wheelsets of the train since the beam has already experienced excitation from all of the previous wheelsets.

- When the graph has been viewed, close the graph

## Peak Acceleration Response of All Train Bodies

In the previous graphs the acceleration response of the sprung masses representing the bodies of the first and last wheelsets representing the train were inspected. The peak vertical acceleration response of all of the sprung masses representing the bodies of the train is now inspected for the 25 kph to 600 kph speed range.



- Open the IMDPlus Moving Mass Analysis Control dialog through the menu or click on the  button in the toolbar.

- Ensure that the Moving mass input and Mode/damping control sections of the dialog are unchanged from the previous analyses
- Enter the Minimum Speed as **6.944**, the Maximum Speed as **166.667** and the Speed Increment as **6.944** to analyse a range of speeds between 6.944 m/s (25 kph) and 166.667 m/s (600 kph) in increments of 6.944 m/s (25 kph).
- Ensure the Quiet time after passage of load is **5**
- Ensure the **IMDPlus determining time step (Nyquist)** option is deselected and the Solution time step is **2E-3**

- Click the **Next** button to proceed. When asked whether the speed range is correct choose **Yes** to continue. When prompted about missing total mass due to mass participation factors below 90% choose **Yes** to continue.

- Ensure **Spring-mass system** is selected
- Set Extent to **All**
- Ensure the Result is set to **Sprung mass 1**
- Ensure the Entity is set to **Acceleration**
- Deselect **Response time history**
- Select **Peak response summary** and **Absolute**
- Ensure all of the selections in the **Output control** are identical to the previous analysis
- Click the **Finish** to proceed.
- Click **Yes** when asked whether to free up disk space by deleting the temporary files created by IMDPlus.

**IMDPlus Moving Mass Output Control**

Node/element selection

Node  Element Entity: Displacement

Extent: Selected Component: RSLT

Node: 31

Individual items  
 Sum of chosen items  
 Average of chosen items

Spring-mass system selection

Spring-mass system Result: Sprung mass 1

Extent: All Entity: Acceleration

System: 32 (-195.0) CRH Tr

Output control

Response time history

Peak response summary  Positive/negative  Envelope

Secondary response spectra (SRS)  Absolute

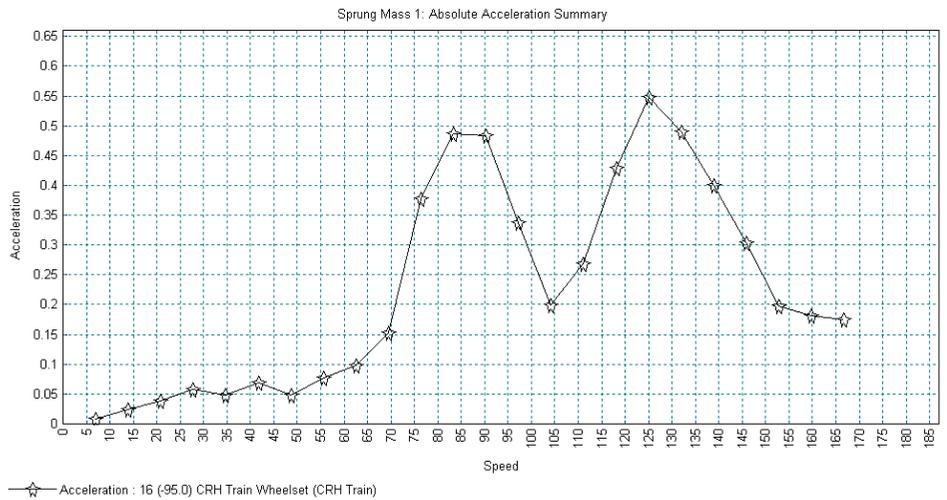
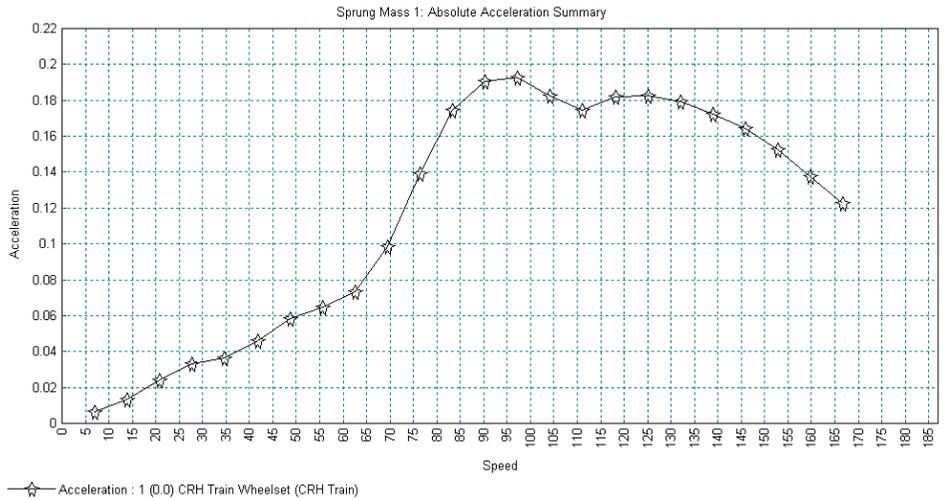
Modal combination/factor history

Generate graphs in Modeller Speed to graph: 1:6.944

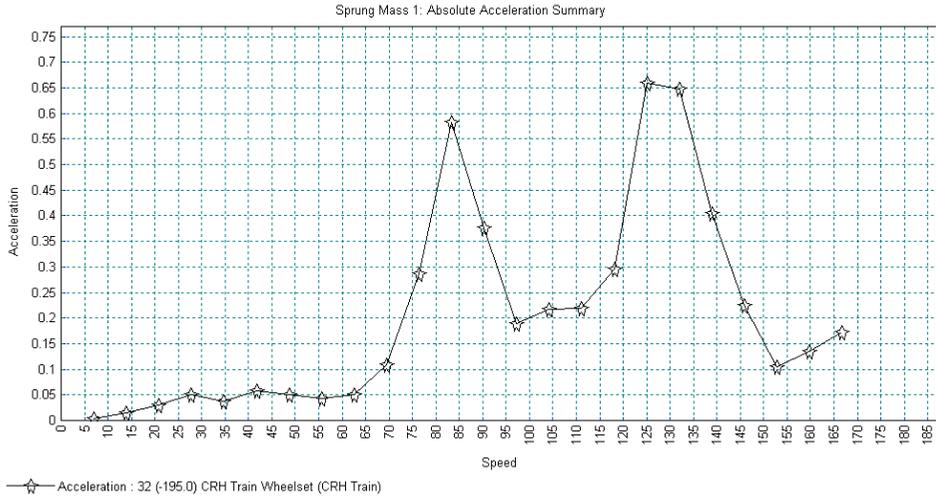
Generate textfile output

< Back Finish Apply Cancel Help

IMDPlus will perform the analysis and the graphs for all 32 of the spring-mass systems will be displayed and can be printed out. A sample of the graphs is shown below demonstrating the different behaviours of each of the sprung bodies of the train (first spring-mass system, sixteenth spring-mass system and the last spring-mass system).



# High Speed Train Modelling through Sprung Masses



The graphs of the absolute peak vertical acceleration of the sprung bodies over the speed range appear to indicate that the peak acceleration increases as the number of wheelset spring-mass systems which have already passed along the beam increases. This is roughly true but from inspection of each wheelset spring-mass system it can be seen that the absolute peak acceleration out of the whole 32 wheelsets of the train actually occurs at the 30<sup>th</sup> wheelset, indicated in the graph below.

For the 30<sup>th</sup> wheelset the absolute peak acceleration is observed at the first resonant frequency equivalent to a train speed of 300 kph and is approximately  $0.78 \text{ m/s}^2$  or almost 0.08 g.



- Close all of the graph windows.

### **Save the model**

File  
Save...

Save the model.

This completes the example.

